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EDITORIAL

The Size of an Electron and the Nature of its Mass

IN the editorial of March, 1944, on "a problem of two electrons and Newton's third law," we showed the importance of the toroidal distribution of displacement currents and the associated magnetic field produced whenever an electron is moving. This magnetic field represents a certain amount of energy which must be expended in accelerating the electron, just as energy must be expended in accelerating a mass. In both cases the energy imparted to the

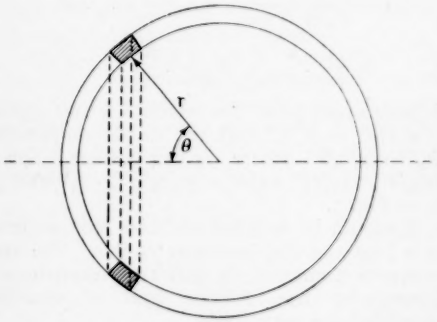

Fig. 1.

moving charge or mass is proportional to the square of the velocity, and one can regard the magnetic field of the moving electron as endowing it with a certain equivalent mass.

The question that then arises is whether the electron has any other mass, that is, whether it has any gravitational mass or whether its entire apparent mass is inertial and due to its magnetic field. If we could isolate an electron and apply the callipers to it we could soon answer this question, for an electron is known to have an apparent mass of 9.11×10^{-28} gramme and a charge of 4.803×10^{-10} electrostatic unit, and if we knew its diameter we could calculate the equivalent mass of its magnetic field. We propose, however, to adopt the reverse procedure and determine the size of the electron on the assumption that its apparent mass is due entirely to the inertia of the magnetic field.

The magnetic field strength H at any point due to a current element $i \, . \, ds$ is equal to $i \, . \, ds \sin \theta / r^2$ where i is in e.m. units. Putting $i \, . \, ds = qv/c$ where q is in e.s. units and $c = 3 \times 10^{10}$, we have $H = qv \sin \theta / cr^2$. The energy in the elemental zone shown in Fig. 1 will be

$$\frac{H^2}{8\pi} \, . \, r d\theta \, . \, 2\pi r \sin \theta \, . \, dr$$

$$= \frac{q^2 v^2}{c^2 r^4} \cdot \frac{dr}{8\pi} \cdot 2\pi r^2 \, . \, \sin^3 \theta \, . \, d\theta$$

$$= \frac{q^2 v^2}{4c^2} \cdot \frac{dr}{r^2} \cdot \sin^3 \theta \, . \, d\theta$$

The energy in the spherical shell will be the integral of this from 0 to π, which, since

$$\int_0^{\pi} \sin^3\theta \, . \, d\theta = 4/3 \quad \text{is} \quad \frac{1}{3} \cdot \frac{q^2v^2}{c^2r^2} \, . \, dr \text{ ergs.}$$

If the electron be regarded as a sphere of radius r_0 the total energy of the magnetic field external to the electron is

$$\frac{1}{3}\frac{q^2v^2}{c^2}\int_{r_0}^{\infty}\frac{dr}{r^2} = \frac{1}{3}\cdot\frac{q^2v^2}{c^2}\cdot\frac{1}{r_0}$$

If we neglect any magnetic field within the electron, which is the same as assuming its charge to be on its surface, this will represent the total energy, which we can therefore equate to $0{\cdot}5mv^2$. Putting

$$0.5\, mv^2 = \frac{1}{3}\cdot\frac{q^2v^2}{c^2}\cdot\frac{1}{r_0}$$

gives

$$r_0 = \frac{2}{3}\cdot\frac{q^2}{mc^2} = \frac{2}{3}\cdot\frac{4.803^2\times 10^{-20}}{9.11\times 10^{-28}\times 9\times 10^{20}}$$
$$= 1.88\times 10^{-13} \text{ cm.}$$

If the charge is not assumed to be confined to the external surface of the electron but is, for simplicity, assumed to be uniformly distributed throughout its volume, the external magnetic energy will be unaffected, but there will now be some internal magnetic energy which will materially affect the result. Outside the electron we were only concerned with displacement currents but inside the electron we have both convection currents due to the moving charge and displacement currents due to the changing electric field.

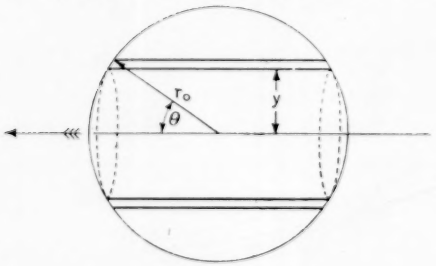

Fig. 2.

If Fig. 2 represents the spherical electron, the density of the charge will be $3q/4\pi r_0^3$, the corresponding current density will be $3qv/4\pi cr_0^3$, and the current linked by a circle of radius y will equal $(3q/4\pi r_0^3) \times \pi y^2 \times v/c$.

This would equal $H \times 2\pi y/4\pi$ where H is the strength of the magnetic field at this radius, if this were the only current, but we must now consider the distribution of displacement current throughout the sphere.

The charge within a sphere of radius r is qr^3/r_0^3 and the radial electric field $\mathfrak{E}$ at this radius is qr/r_0^3. In Fig. 3 $\mathfrak{E}$ at a fixed point P, for which $\theta = \pi/2$, is plotted for two successive positions of the source (assumed positive), *i.e.*, of the centre of the electron, and it can be seen that $\dfrac{d\mathfrak{E}/dt}{\mathfrak{E}} = \dfrac{v}{r}$.

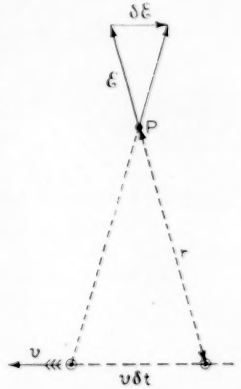

Fig. 3.

If i_d is the displacement current, we have therefore

$$i_d = \frac{dD}{c\,.\,dt} = \frac{1}{4\pi c}\cdot\frac{d\mathfrak{E}}{dt} = \frac{1}{4\pi c}\cdot\frac{v}{r}\cdot\mathfrak{E}$$
$$= \frac{1}{4\pi c}\cdot\frac{v}{r}\cdot\frac{qr}{r_0^3} = \frac{1}{4\pi c}\cdot\frac{qv}{r_0^3}$$

At any other point for which θ is not equal to $\pi/2$, $\mathfrak{E} = qr/r_0^3$ and the axial component $\mathfrak{E}_x = qx/r_0^3$ where $x = r\cos\theta$. Since $d\mathfrak{E}_x/dt = (q/r_0^3)\, dx/dt = qv/r_0^3$, $i_d = qv/4\pi cr_0^3$ as in Fig. 3.

It should be noted that this is independent of r; so the displacement current, like the convection current, is uniformly distributed throughout the volume and of exactly one-third the magnitude.

It can be seen at once from Fig 3 that the displacement current is in the opposite direction to the convection current and that the resultant current density is $2qv/4\pi cr_0^3$. Hence

$$\frac{H\times 2\pi y}{4\pi} = \frac{2qv}{4\pi cr_0^3}\times\pi y^2$$

and $H = \frac{qvy}{cr_0^3}$ anywhere inside the sphere at a distance y from the axis. It is of interest to note that on the surface of the sphere $x = r_0 \sin\theta$ and $H = qv \sin\theta/cr_0^2$ as found for the external field.

Having found the value of H throughout the sphere we must now find the magnetic energy. The volume of a cylindrical shell of radius y and thickness dy is $2\pi r_0 \sin\theta \times 2r_0\cos\theta \times r_0\cos\theta\, d\theta = 4\pi r_0^3 \sin\theta\cos^2\theta\, d\theta$ where $y = r_0\sin\theta$.

Multiplying this by $H^2/8\pi$ we have

$$\frac{q^2v^2\sin^2\theta}{c^2r_0^4} \times \frac{4\pi r_0^3 \sin\theta\cos^2\theta\, d\theta}{8\pi} = \frac{q^2v^2}{2c^2r_0}\sin^3\theta\cos^2\theta\, d\theta$$

For the whole sphere $\int_0^{\pi/2} \sin^3\theta\cos^2\theta\, d\theta$

$$= \int_0^{\pi/2} (\sin^3\theta - \sin^5\theta)\, d\theta$$

$$= \tfrac{2}{3} - \tfrac{8}{15} = \tfrac{2}{15}$$

and thus the whole magnetic energy inside the sphere is equal to $\frac{q^2v^2}{15\,c^2r_0}$ which is exactly a fifth of the external magnetic energy. The value calculated above for the radius r_0 by equating $0.5\ mv^2$ to the magnetic energy should therefore be increased from 1.88×10^{-13} to

$$\tfrac{6}{5} \times 1.88 \times 10^{-13} = 2.256 \times 10^{-13}\ \text{cm.}$$

This is based on the purely arbitrary assumption that the charge distribution is uniform throughout the assumed sphere.

Although only a sixth part of the total energy is within the electron, the greater part is concentrated within a very small radius. The energy contained in the space between the surface of the electron and a spherical surface of twice the radius will be equal to $\frac{1}{3} \cdot \frac{q^2v^2}{c^2}\left(\frac{1}{r_0} - \frac{1}{2r_0}\right)$ *i.e.*, to $\frac{1}{6}\frac{q^2v^2}{c^2r_0}$ so that, with the internal energy, we can say that about 0.6 of the total magnetic energy is contained within a sphere of twice the electron radius.

We would emphasize that the sole object of this calculation is to determine the size that an electron must have if its apparent mass is due solely to the energy of the magnetic field set up by its motion. If its radius can be proved to have some greater value, then the above calculation will show to what extent its total mass can be ascribed to its magnetic field and to what extent it possesses gravitational mass.

It is interesting to consider how its magnetic field endows the electron with all the kinetic properties that one usually associates with mass, such as momentum, kinetic energy and centrifugal force. To accelerate it the magnetic field must be increased and this induces an electric field opposing the motion of the electron ; when it decelerates, the magnetic field decreases and the induced electric field tends to keep the electron in motion ; thus any departure from uniform velocity in a straight line calls into being forces opposing the change. In considering the rotation of an electron around an atomic nucleus we can thus apply the usual Newtonian laws on the assumption that the electron has ordinary gravitational mass even if it really has no such mass ; surely a very striking and unexpected conclusion. G. W. O. H.

Correction.

In the Editorial of September last we referred to Dr. Geoffrey Builder as " of the D.S.I.R. of Australia." We learn, however, that Dr. Builder is no longer connected with the Radio Research organization, but has been for some time in practice in Australia as a consulting engineer.

TRANSIENT RESPONSE OF FILTERS*

By D. G. Tucker, Ph.D., A.M.I.E.E.

(*Post Office Research Station*)

SUMMARY.—It is shown that the normal treatments of the transient response of filters are either not sufficiently simple or are not really suitable for all types of filter; the method which considers the filter as an idealized transmission characteristic is satisfactory for multi-section filters and gives results in simple terms, but for single-section filters a special treatment is desirable. A method of determining the build-up and decay characteristics is given for a single section of the "6-element symmetrical" band-pass filter, and the type of equation obtained is of general application to other types of section.

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Bibliography.

Introduction

THE question of the transient response of filters is extremely complicated, and compared with steady-state treatments involves advanced mathematics from whatever point of view it is approached. It is very important in pulse-signalling systems, however, and consequently certain "rules of thumb" have been commonly quoted, and used in design. The two important ones are:

(*a*) The build-up time of a band-pass filter for an applied signal of mid-band frequency is equal to the reciprocal of the bandwidth in cycles/sec. For a low-pass filter, the build-up time of an applied D.C. signal is one-half of the reciprocal of the cut-off frequency in cycles/sec.

(*b*) The amplitude of the peak of the transient caused by the sudden application or removal of a frequency outside the pass-band of a band-pass filter is proportional to the bandwidth and inversely proportional to the difference between applied frequency and mid-band frequency.

Various text-books[1,2,3] on filters and circuits give some treatment of this problem and quote or justify the above rules. The method of approach is to consider the filter not as a combination of impedance elements but as an idealized transmission characteristic in which the pass-band is quite flat, the rate of cut-off infinite, and the attenuation outside the pass-band infinite; the phase characteristic is considered linear throughout the pass-band. It is not difficult to appreciate that this hardly represents a practical filter of conventional type. It is worth noting, however, that substantially "ideal" filters can be made, see Bode and Dietzold[5] and Levy[6].

Carson and Zobel[4] give a treatment which is based on the actual circuits, but assumes smooth terminations on an image impedance basis. This treatment is extremely complicated, and gives results which are difficult to use.

Some of the results of these two methods will be briefly summarized in subsequent sections and their shortcomings discussed.

Another method of computing the transient response of any kind of band-pass filter is given by E. C. Cherry[17], who uses measured or calculated values of the attenuation and phase-shift of the filter at every frequency component of the applied *pulsed* tone (i.e., tone regularly applied and removed). The method therefore gives only an approximation (though generally quite adequate) to the response to a single unit-step envelope. It is, moreover, rather laborious. It has the very great advantage, however, that it enables the envelope shape to be determined for applied frequencies different from the

* MS. accepted by the Editor, September 1945.

mid-band frequency yet still within the pass-band.

It has been found in practice that the first method of treatment gives results which are not only simple to compute, but also represent the performance of real filters, provided the filters have sufficient sections to make their performance approach the ideal in respect of rapid cut-off and high attenuation outside the pass band. Such performance may be attained by filters of $1\frac{1}{2}$ or 2 sections (or more), and, as will be shown in due course, the analytical results apply very closely in all major respects.

For filters of a single section, however, the theory is not adequate. Large discrepancies in performance are found. A special treatment of this case has therefore been developed, and is described in Section 3. The analysis is based on the actual filter circuit, and equations are given for the build-up and decay of pulses of the mid-band frequency of band-pass filters.

Throughout the paper, only band-pass filters are discussed. This is the most general case, and for most purposes the performance of a low-pass filter can be taken as equivalent to that of a band-pass filter of bandwidth equal to twice the low-pass cut-off frequency, and with a mid-band frequency of zero*. Thus the pulse envelopes discussed for band-pass filters represent the actual current for low-pass filters.

Again, only the "6-element" series-derived T-section is considered in dealing with single-section filters. The steady-state responses of the various arrangements of "6-element" (or symmetrical) sections are different, and are fully discussed in a previous paper,[11] but it is unlikely that the transient responses will be greatly different, and it is thought that the one case treated should be adequate. Unsymmetrical filters (i.e. filters with different steady-state responses above and below the pass-band) are not dealt with here, but have been found to be quite capable of treatment by the methods indicated.

It must be emphasised that all the methods discussed for band-pass filters *assume that the bandwidth is small* in relation to the mid-band frequency. There is no corresponding limitation in the application to low-pass filters.

* This means that in the equations given, $n\omega_0$ for band-pass filters should be replaced by $2\omega_c$ for low-pass filters, where ω_c = cut-off angular frequency.

List of the more Important Symbols

A = attenuation ratio of pass-band in "ideal" filter.
E = peak amplitude of applied signal.
G = a constant in the transient term for a single-section filter (and is a function of m).
$M = \frac{m}{n}\left(\frac{p}{\omega_0} + \frac{\omega_0}{p}\right)$
m = derivation parameter of filter.
n = fractional bandwidth = bandwidth/mid-band frequency.
p = differential operator = d/dt.
t = time in seconds.
R = design resistance of filter.
W = bandwidth in radians per sec.
$y = p/\omega_0$.
ω_0 = mid-band angular frequency.
ω_c = cut-off angular frequency of low-pass filter.

1. Analysis of Band-Pass Filter with an Idealized Transmission Characteristic

The filter is assumed to have a flat attenuation response in the pass-band, infinitely rapid cut-off, infinite attenuation in the attenuating bands, and a linear phase characteristic in the pass-band. The applied signal consists of a voltage $E \cos \omega_1 t$ suddenly switched on at time $t = 0$. The envelope of this signal is the Heaviside Unit Step. The signal can be expressed as the Fourier integral:

$$e_1 = \frac{E}{2}\cos\omega_1 t + \frac{E}{2\pi}\int_0^\infty \frac{1}{\omega_1 - \omega}\sin\omega t \,.\, d\omega + \frac{E}{2\pi}\int_0^\infty \frac{1}{\omega_1 + \omega}\sin\omega t \,.\, d\omega \qquad (1)$$

where e_1 = voltage at any instant,
$E \cos \omega_1 t$ = suddenly impressed wave
ω = any angular frequency in the spectrum.

Alternatively, the following form may be used if ω is taken to be the envelope component, i.e., the difference between any frequency being considered and the applied frequency:

$$e_1 = \frac{E}{2}\cos\omega_1 t - \frac{E}{2\pi}\int_0^\infty \frac{1}{\omega}\sin(\omega_1 - \omega)t \,.\, d\omega + \frac{E}{2\pi}\int_0^\infty \frac{1}{\omega}\sin(\omega_1 + \omega)t \,.\, d\omega \quad .. \quad (2)$$

This signal is restricted in the limits of integration by postulated filter character-

istics. If the cut-off frequencies are $\omega_0 + W/2$ and $\omega_0 - W/2$, where ω_0 = mid-band (angular) frequency and W is the bandwidth (angular frequency), then the integration has to be performed between the limits $\omega_0 - W/2$ and $\omega_0 + W/2$. Most of the working is given by Guillemin[3], and leads to the following results :—

1.1 *Impressed Frequency = Mid-band Frequency* $= \omega_0$.

Output voltage, $e_2(t)$

$$= EA\left[\tfrac{1}{2} + \frac{1}{\pi} Si \,.\, \frac{W}{2}(t - t_0)\right] \cos \omega_0 t \quad (3)$$

where A = attenuation ratio of pass-band,

t_0 = transmission time (i.e., time from the instant of application of the input to that instant when the output has half its steady amplitude)

$= \theta/(\omega - \omega_0)$ for any frequency in the pass-band where θ = phase-shift.

Si = " Sine-integral," defined by

$$Si(x) = \int_0^x \frac{\sin u}{u} \,.\, du$$

Build-up time (defined as the time required for the amplitude to change from zero to the steady-state value at a rate corresponding to that at the half-amplitude point),

$$= \frac{EA}{\left[\frac{de_2}{dt}\right]_{t=t_0}} = \frac{EA}{\frac{EA}{\pi} \,.\, \left[\frac{d}{dt} Si \,.\, \frac{W}{2}(t - t_0)\right]_{t=t_0}}$$

The differential coefficient required is of the form

$$\frac{d}{dx} \,.\, \int_0^{kx} f(u) \,.\, du$$

and this can be shown to be equal to

$$k \,.\, f(kx)$$

$$\therefore \quad \frac{d}{dt} \,.\, Si \,.\, \frac{W}{2}(t - t_0) = \frac{W}{2} \,.\, \frac{\sin \frac{W}{2}(t - t_0)}{\frac{W}{2}(t - t_0)}$$

and when $t = t_0$, this is $W/2$.

$$\therefore \quad \text{Build-up time} = \frac{2\pi}{W} \quad .. \quad .. \quad (4a)$$

Note that W is the bandwidth in rads./sec.

$$\therefore \quad \text{Build-up time} = \frac{1}{\text{Bandwidth in cycles/sec.}} \quad .. \quad .. \quad (4b)$$

1.2 *Impressed Frequency NOT equal to Mid-band Frequency*

(*a*) The case where the impressed frequency lies in the pass-band but is not equal to the mid-band frequency is difficult and is not fully discussed in the text-books quoted. (But see Cherry[17].)

(*b*) Where the impressed frequency lies well outside the pass-band, the voltage produced at the output of the filter is shown by Guillemin to be

$$e_2(t) = \frac{EAW\mu}{\pi(\omega_0^2 - \omega^2)} \cdot \frac{\sin \frac{W}{2}(t - t_0)}{\frac{W}{2}(t - t_0)} \,.$$

$$\sin (\omega_0 t + \psi) \quad .. \quad .. \quad (5)$$

where ω = impressed angular frequency

$$\mu = \sqrt{\omega_0^2 \cos^2 \alpha + \omega^2 \sin^2 \alpha}$$

$$\psi = \tan^{-1}\left(\frac{\omega}{\omega_0} \tan \alpha\right)$$

α = initial phase angle of applied pulse, $E \cos(\omega t + \alpha)$. Now if $W \ll \omega_0$ (i.e. a narrow bandwidth) and ω/ω_0 is not very different from unity (conditions which frequently apply in practice) then

$$\mu = \omega_0 \qquad \psi = \alpha$$

$$(\omega_0^2 - \omega^2) = 2\omega_0(\omega_0 - \omega)$$

$$\therefore \quad e_2(t) = \frac{EAW}{2\pi(\omega_0 - \omega)} \cdot \frac{\sin \frac{W}{2}(t - t_0)}{\frac{W}{2}(t - t_0)} \,.$$

$$\sin (\omega_0 t + \alpha) \quad .. \quad .. \quad (6)$$

This represents an envelope of *maximum amplitude* at $t = t_0$ given by

$$e_{2 \cdot \max} = \frac{EA}{2\pi} \cdot \frac{W}{(\omega_0 - \omega)} \quad .. \quad .. \quad (7)$$

which gives the rule that the transient peak is proportional to bandwidth and inversely proportional to the frequency difference between the applied pulse and mid-band.

The envelope is itself oscillatory, on both sides of $t = t_0$, and decays fairly rapidly.

An important point to be noted is that when ω is well removed from ω_0, then the transient peak amplitude depends on the

initial phase angle, α. We see that (from (5))

$$\frac{\text{Transient peak for } \alpha = 0}{\text{Transient peak for } \alpha = \pi/2} = \frac{\omega_0}{\omega} \quad .. \quad (8)$$

which may be quite a large or small ratio. It is interesting to note that the same relationship is obtained with a single tuned circuit[18]. However, when the bandwidth n is very small, this effect is often unimportant.

1.3 *Comments on the Theory*

The theory outlined above is very useful in that certain simple results are obtainable which may be quite adequate for many practical purposes. The simple rules quoted at the beginning of the section have been found to agree with practical measurements provided the filter has enough sections to give a rapid cut-off with an adequate attenuation outside the pass-band.

An example of how the relation of Equ. (7) corresponds with practice is shown in Fig. 1. A half-section filter ($m = 0.858$, $\omega_0 = 16{,}650$, $n = \frac{W}{\omega_0} = 0.037$) gives a transient peak about 9 db larger than that given by Equ. (7). A

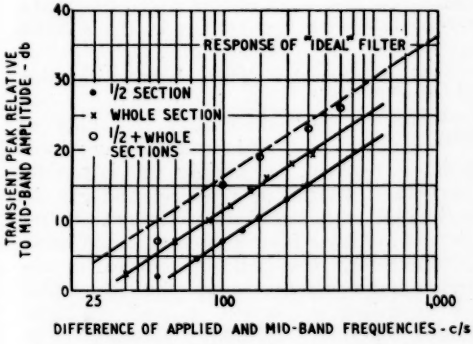

Fig. 1. *Transient response of 6-element filters; bandwidth = 100 c/s, mid-band = 2,650 c/s.*

whole-section filter ($m = 0.884$, $n = 0.037$) gives a transient peak 5 db larger. The two filters together agree with equation (7) to within one or two decibels. Generally speaking, for narrow-band filters of symmetrical 4- or 6-element types it has been found that equation (7) applies quite closely for a number of sections in excess of $1\frac{1}{2}$ or 2.

Regarding build-up time, and defining this as in Sect. 1.1, tests made on the same filter sections* as before gave these results:—

$\frac{1}{2}$ section, $m = 0.858$,
build-up time = 9 ms

1 section, $m = 0.884$,
build-up time = 9.5 ms

These two together,
build-up time = 10.0 ms

Also, 1 section, constant-k,
build-up time = 8.5 ms

Build-up time from equation (4)
= 10.0 ms

Thus the better filter, which gave agreement on the transient peak amplitude, also gives agreement on build-up time. The other filters give a shorter build-up time, but the discrepancy is small—under 15 per cent.

Regarding the actual waveforms of the transient components, given by equations (3) for $\omega = \omega_0$ and by equations (5) and (6) for $\omega \neq \omega_0$, and illustrated by Guillemin (see Vol. II, Figs. 207 and 208), it is found that practical filters give a rather different result. For instance, equation (3) gives some output, with an oscillatory envelope, right from negative time up to the actual build-up; it gives, too, an oscillatory component to the envelope after build-up has occurred. This is because of the characteristics of the sine-integral (tabulated in numerical form in certain mathematical tables, notably Jahnke and Emde), which has an oscillatory component. In practice, no such preliminary oscillation is obtained, although an oscillation after build-up can often be observed. This will be clear from the oscillograms accompanying this paper. Again, equations (5) and (6) give the preliminary output, which is not found in practice.

As far as the shape of the main part of the build-up and decay curves is concerned (dealing with the case of applied frequency = mid-band frequency), equation (3) gives the build-up envelope as

$$\tfrac{1}{2} + \frac{1}{\pi} Si \cdot \frac{W}{2}(t - t_0) \quad .. \quad .. \quad (3a)$$

where t_0 is the time at which the envelope reaches half its steady-state amplitude, which is here taken as unity. A graph of this function over the main part of its range is

* All these filters had a design bandwidth ($n\omega_0$) of 100 c/s. Throughout this work, "bandwidth" means the quantity $n\omega_0$, and is not related to any arbitrary attenuation values, as is sometimes the case in other works.

shown by the full-line curve of Fig. 2, the values of the sine-integral being obtained from Jahnke and Emde's Tables.

The build-up and decay curves of three actual filters (measured on a Cambridge Oscillograph) are compared with this " ideal " curve in the same figure. No account is taken in this instance of the " transmission time " ; all curves take as zero time the time at which the half-amplitude is reached. It is clear that the curve of the single-section constant-k filter is considerably different from (3a) ; it is fitted considerably more closely by the equations (22) and (23), to be discussed later in Section 3.2. A derived section is still more unsymmetrical about the half-amplitude point, and consequently equation (3a) is an even poorer representation of its characteristics. But the build-up and decay of the $1\frac{1}{2}$-section filter are simulated reasonably closely by the sine-integral curve, which lies about midway between the later halves of the build-up and decay envelopes, which are different in that the decay shows an overshoot or oscillation, as does the sine-integral. The build-up of a 2-section filter errs in some respects in the opposite way from the 1-section filter ; it has a slower build-up than the sine-integral curve indicates, but *is quite symmetrical about the half-amplitude point.* No doubt, the presence of dissipation in the filter components accounts for some discrepancies, but, at any rate, the filters used in the experiments had a value of $Q \times n$ equal to about 4, a value which is typical for narrow-band filters.

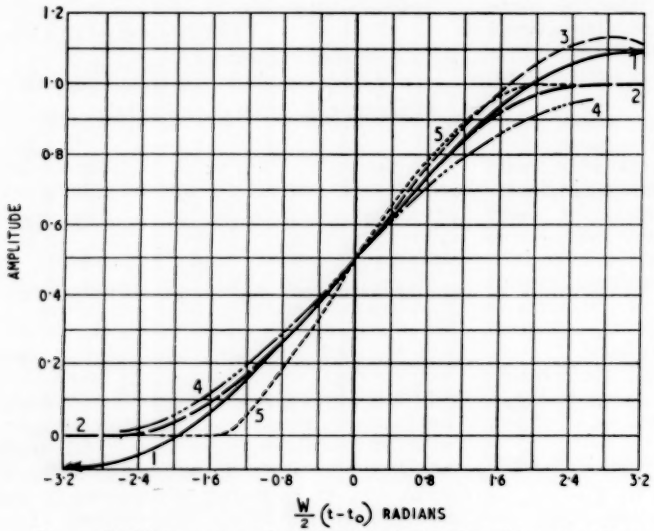

Fig. 2. *Build-up curves of " ideal " and actual multi-section filters. The upper end of curve* 1 *is oscillatory about amplitude* 1.0, *and the lower end about* 0.

Curve 1. [*Sine integral* $\times\ 1/\pi$] $+\ \frac{1}{2}$.
2. *Measured build-up* } 1-*sect.*, $m = 1$, + $\frac{1}{2}$-*sect.*, $m = 0.884$.
3. *Measured decay* }
4. *Measured build-up*, 1-*sect.*, $m = 1$, + 1-*sect.*, $m = 0.884$.
5. *Measured build-up*, 1-*sect.*, $m = 1$, *alone.*

It is to be noted that as the number of sections is increased beyond two, the build-up characteristics are not much affected if the sections are matched on an image impedance basis and are non-dissipative, because then the addition of further sections does not greatly affect the pass-band loss-frequency response. But if the filter is dissipative or if the sections are mutually independent and each terminated in fixed resistances, then the build-up time is increased as further sections are added, due to the reduction in the *effective* bandwidth.

The above discussion has ignored the actual time after switching at which the half-amplitude point is reached. Generally this is not an important matter in practice, and can be estimated with sufficient accuracy from the slope of the phase-frequency curve at mid-band. Alternatively, the build-up curves of the individual sections may be calculated as described in Section 3, and the times added together. If greater precision is required, the Superposition method should be adopted.

It may be concluded here that the " ideal " theory is reasonably satisfactory for multi-section filters, but not for single sections.

Some further comment on some of these points is given by Bell.[7]*

A very fine paper on the idealised filter has been published by Kamphausen[8], and this shows—

(*a*) the effect of short pulses, where the " make " and " break " transients interfere,

* See footnote on p. 42.

and where the maximum transient amplitude may be doubled.

(*b*) the effect of extremely short pulses (duration $< \frac{1}{\omega_0 - \omega}$) where the transient is independent of ω_0 and ω.

(*c*) the effect of applying pulses with an exponential envelope. This case arises frequently in practice where the pulse may be transmitted through a sending-end filter or tuned circuit (which gives it an envelope with a shape not far from exponential) before being applied to a receiving-end filter.

2. Analysis of Band-Pass Filter by Carson and Zobel's Method

This method of treatment of the problem assumes an infinite succession of filter sections, all the same, so that all junctions are reflectionless. An expression is then derived for the current in the nth section. The working involves advanced mathematics, a discussion of which is outside the scope of this paper; consequently, only a summary of the main results will be given, all based on the work of Carson and Zobel[4]. Only the case of a constant-k band filter will be discussed.

The indicial admittance up to the nth section* is

$$A_n(t) = \frac{W}{\omega_0 R_0} J_{2n}\left(\frac{Wt}{2}\right) \sin \omega_0 t \quad .. \quad (9)$$

where $J_{2n}(\frac{Wt}{2})$ = Bessel function of order $2n$ and argument $\frac{Wt}{2}$.

R_0 = design impedance.

Now, $A_n(t)$ is very small until $\frac{Wt}{2} \geqslant 2n$. Thus the signal is transmitted with an apparent velocity of propagation of approximately $W/4$ sections per second.

* i.e., the current flowing in the nth section in response to a unit voltage applied at the input of the filter.

For values of $\frac{Wt}{2} > 2n$, the character of the function is indicated by the following approximate formulae:—

$$A_n(t) \approx \frac{W}{2\omega_0 R_0} h_{2n} \sqrt{\frac{4}{\pi Wt}} \cdot \left[\sin\left(\omega_0 t - q_{2n} \cdot \frac{Wt}{2} + \theta_{2n}\right) + \sin\left(\omega_0 t + q_{2n} \cdot \frac{Wt}{2} - \theta_{2n}\right)\right] \quad .. \quad .. \quad (10)$$

where $h_{2n} = \left[\frac{1}{1 - \frac{16\,n^2}{W^2 t^2}}\right]^{\frac{1}{4}}$

$$q_{2n} = \sqrt{1 - \frac{16\,n^2}{W^2 t^2}}$$

and $$\theta_{2n} = \frac{4n+1}{4}\pi - 2n \sin^{-1}\left(\frac{4n}{Wt}\right)$$

We see from equation (10) that the transient signal is composed of two frequencies equally spaced around the mid-band frequency. But, these two frequencies vary with time, owing to the factor q_{2n}, and lie within the pass-band, approaching the cut-off frequencies when t is large.

So, ultimately

$$A_n(t) \to \frac{W}{2\omega_0 R_0} \sqrt{\frac{4}{\pi Wt}} \left[\sin\left(\omega_0 t - \frac{Wt}{2} + \frac{4n+1}{4}\pi\right) + \sin\left(\omega_0 t + \frac{Wt}{2} - \frac{4n+1}{4}\pi\right)\right] \quad .. \quad .. \quad (11)$$

When an e.m.f. $\sin(\omega t + \theta)$ is applied to the filter, the build-up of current in the nth section is given by

$$\frac{1}{R_0}\left\{\sin(\omega_0 t + \theta) \int_0^{\frac{Wt}{2}} J_{2n}\left(\frac{Wt_1}{2}\right) \cdot \cos\left[(\omega - \omega_0)(t - t_1)\right] \cdot d\left(\frac{Wt_1}{2}\right) + \cos(\omega_0 t + \theta) \int_0^{\frac{Wt}{2}} J_{2n}\left(\frac{Wt_1}{2}\right) \sin\left[(\omega - \omega_0)(t - t_1)\right] \cdot d\left(\frac{Wt_1}{2}\right)\right\} \quad .. \quad .. \quad (12)$$

where t_1 is a variable of integration only.

The derivation of this expression is not given by Carson and Zobel, except that it is obtained by substituting the indicial admittance into the Superposition Theorem. The form of this build-up characteristic is extremely inconvenient, but the integrals can be computed, and typical curves of the

build-up at various frequencies are given by Carson and Zobel.

The time T required for the alternating current to build-up to its steady-state value, the applied frequency being within the bandwidth, is given approximately by

$$T = \frac{4n}{W} \cdot \frac{1}{\left[1 - 4\left(\frac{\omega - \omega_0}{W}\right)^2\right]^{\frac{1}{2}}} \quad \cdots \quad (13)$$

This is, of course, the time from $t = 0$, and includes the "transmission-time" of the filter (referred to as t_0 in section 1.1). When $\omega = \omega_0$, the time is $\frac{4n}{W}$, corresponding to the velocity of propagation of $W/4$ sections per second.

One very definite conclusion to be reached from Carson and Zobel's work is that the transient oscillations in the filter are of frequency located in the bandwidth, *varying with time*, and settling down to the cut-off frequencies. This is rather different from the result of the previous method of treatment (Section 1), which gives *fixed* transient frequencies of the cut-off value. Equations 3, 5 and 6 all give transient frequencies $\omega_0 \pm \frac{W}{2}$; that this is so in the case of equation (3) may be seen from its differential coefficient with respect to t.

Carson and Zobel's method does not give any simple expressions for the rate of build-up at the half-amplitude point. The expression (13) gives the "transmission time" at mid-band $= 4n/W$. The method of Section 1 gives "transmission time" $= 2\theta_w/W$ where θ_w = phase shift at cut-off. Thus the two methods give similar results in this respect, but they are only the same if $\theta_w = 2$ radians per section for a constant-k filter. This is, of course, not the case—the value of θ_w for a constant-k filter under image impedance conditions is π radians per section. It is worth noting, however, that if we consider transmission time as being given by the slope of the phase shift-frequency curve at mid-band, we find this to be $4/W$ for one section, which agrees with Carson and Zobel's result. There still remains a small discrepancy, however; Carson and Zobel define transmission time as the time taken to reach substantially steady-state amplitude. The "ideal" method defines it as the time taken to reach half the steady-state amplitude.

In general, the above method of analysis is far less useful than that involving the idealized transmission characteristics, not because it is less accurate (indeed, it represents real filters more accurately), but because it does not lead to simple and easily-applied relationships. Further work along the lines of this section has been published, as follows:—

Carson and Zobel give an outline treatment of the case of any terminating impedance in Appendix IV of their paper.

Finite, resistance-terminated filters (low-pass type only) are dealt with, again in outline, by Weber and Di Toro[9].

Finite, resistance-terminated, dissipative filters (low and high-pass types) are dealt with in reasonable detail, with experimental results, by Chu and Chang[10].

(To be concluded)

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* Bell comments on some of the difficulties associated with the "ideal" theory, but it seems doubtful whether his suggestions as to its real significance are valid. For instance, his main conclusion that the ideal theory really represents a half-section filter is not supported by experimental evidence, and the present author has shown that, in practical cases, it is only filters of more than one section which are reasonably well represented by the "ideal" theory, and that smaller filters require special analysis. Bell points out, of course, that the effect of dissipation in the filter will be to modify the oscillatory components of the transient envelope. He is misleading in his discussion on p. 325, in connection with the term $\frac{1}{2}$ in equation (3) of the present paper. For a band-pass filter, the expression $\frac{1}{2} + \frac{1}{\pi} Si \cdot \frac{W}{2}(t - t_0)$ represents the *envelope* of the wave, so that the $\frac{1}{2}$ presents no more difficulty than for a low-pass filter, which (as previously pointed out) may be regarded as a band-pass filter of pass-bandwidth equal to *twice* its cut-off frequency (i.e., extending from $-f_c$ to $+f_c$) with a mid-band frequency of zero.

PUSH-PULL CIRCUIT ANALYSIS*

Cathode-Coupled Output Stage

By S. W. Amos, B.Sc.(Hons.), Grad.I.E.E.

(*B.B.C. Engineering Training Department*)

MOST circuits used for providing the out-of-phase signals necessary for driving push-pull stages of amplification employ a valve or centre-tapped transformer to perform the " phase-splitting " but the circuit given in Fig. 1 requires neither of these components. This circuit is one in which one output valve derives its input by cathode-coupling from the other. In its application to resistance-coupled intermediate A.F. stages it is quite well-known[1,2] but its suitability for stages having a transformer output coupling is less widely recognised. It has, however, been used in this way by C. G. Mayo and H. D. Ellis in the design of certain B.B.C. audio-frequency amplifiers.

The input signal is applied to the terminals $A\,B$, of which B is earthed, and the two valves V_1 and V_2 (usually high-slope tetrodes or pentodes, but drawn as triodes for convenience) operate automatically in push-pull. The phase-reversal is accomplished as follows. Consider the circuit with V_2 and all its grid circuit components removed. The valve V_1 then operates normally but with some current feedback due to the A.F. potentials developed across the unbypassed grid bias resistance R_k and the resistance R_0. These A.F. potentials provide the input signal for V_2 when it is in circuit.

In order that the A.C. component of the anode current of V_2 shall be 180 deg. out of phase with that of V_1, the cathode of V_2 must be joined to the cathode of V_1 and its grid must be joined to earth. This latter connection is actually made via a capacitor in order that the correct value of grid bias may be applied to V_2 by means of the grid leak R_2. Thus the anode currents of both valves pass through R_k (which hence acts as a common grid bias resistance) and R_0 and the amplitudes of the A.C. components of these anode currents adjust themselves so that they are unequal, V_1 providing the greater A.C. component. This difference in amplitude is necessary in order to provide the input signal for V_2. It is the purpose of this analysis to find the effect of varying R_0 on the performance of the circuit and hence to find an expression for its optimum value.

Let i_1 and i_2 = amplitude of the A.C. components of the anode currents of V_1 and V_2 respectively (both currents are regarded as positive, their directions being indicated in Fig. 1, and i_1 is assumed greater than i_2).

E_1 and E_2 = amplitudes of the alternating potentials between grid and cathode of V_1 and V_2 respectively

E_k = amplitude of the P.D. developed across R_k

E_0 = amplitude of the P.D. developed across R_0

E = amplitude of the input signal applied across AB.

From Fig. 1, we have

$$\begin{aligned} E_1 &= E - (E_0 + E_k) = \\ &\quad E - [(i_1 - i_2)R_0 + (i_1 - i_2)R_k] \\ &= E - (i_1 - i_2)(R_0 + R_k) \\ &= E - (i_1 - i_2)R_0' \qquad .. \qquad .. \quad (1) \end{aligned}$$

in which $R_0' = R_0 + R_k$

$$\begin{aligned} \text{Also } E_2 &= E_0 + E_k \\ &= (i_1 - i_2)(R_0 + R_k) \\ &= (i_1 - i_2)R_0' \qquad .. \qquad .. \quad (2) \end{aligned}$$

Regarding V_1 and V_2 as constant current generators, we have

$$i_1 = g_m E_1 = g_m\,[E - (i_1 - i_2)\,R_0'] \quad (3)$$

and

$$i_2 = g_m E_2 = g_m (i_1 - i_2) R_0' \qquad .. \quad (4)$$

in which V_1 and V_2 are assumed to be valves with the same value of g_m. It is considered unnecessary in equations (3) and (4) to include negative signs, thus, $i_1 = -\,g_m E_1$. With the input signals and anode currents of V_1 and V_2 as indicated in Fig. 1, all phase relationships are taken care of by using positive signs throughout as in equations (3) and (4).

From (3) $\quad (i_1 - i_2)R_0' = E - \dfrac{i_1}{g_m}$

* MS. accepted by the Editor, September 1945.

From (4) $(i_1 - i_2)R_0' = \frac{i_2}{g_m}$

$$\therefore \quad E - \frac{i_1}{g_m} = \frac{i_2}{g_m}$$

$$i_1 + i_2 = g_m E$$

which shows that the combined output currents of V_1 and V_2 have the same value that would be given by a single valve with the same value of g_m and the same total input E.

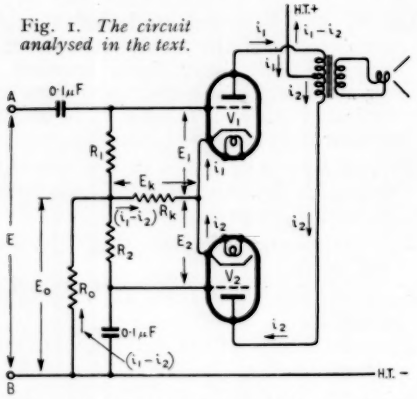

Fig. 1. *The circuit analysed in the text.*

Solving (3) and (4) for i_1 and i_2 we find

$$i_1 = \frac{(1 + g_m R_0')g_m E}{1 + 2g_m R_0'} \quad .. \quad .. \quad (5)$$

and $$i_2 = \frac{g_m^2 R_0' E}{1 + 2g_m R_0'} \quad .. \quad .. \quad .. \quad (6)$$

$$\therefore \quad i_1 - i_2 = \frac{g_m E}{1 + 2g_m R_0'} \quad .. \quad .. \quad (7)$$

In order to assess the difference between i_1 and i_2 we shall use the symbol Δ to represent (difference in amplitude of the A.C. components of the anode currents of V_1 and V_2)/(mean amplitude of the A.C. components)

i.e. $$\Delta = \frac{i_1 - i_2}{\frac{1}{2}(i_1 + i_2)}$$

From the expressions given above for $i_1 - i_2$ and $i_1 + i_2$

$$\Delta = \frac{2g_m E}{1 + 2g_m R_0'} \cdot \frac{1}{g_m E} = \frac{2}{1 + 2g_m R_0'}$$

In most practical problems it will be found that $2g_m R_0'$ considerably exceeds unity, so that

$$\Delta \approx \frac{1}{g_m R_0'}$$

$$\therefore \quad R_0' \approx \frac{1}{\Delta g_m} \quad .. \quad .. \quad .. \quad .. \quad (8)$$

Suppose, for a particular type of valve, $g_m = 10$ mA/volt and we do not wish the difference in A.C. components of the anode currents to exceed 5 per cent. of their mean value. We have, from (8).

$$R_0' \approx \frac{1}{\frac{1}{20} \cdot \frac{10}{1{,}000}} = 2{,}000 \text{ ohms.}$$

R_0' is, of course, the value of total cathode resistance, i.e., grid bias resistance R_k, together with the coupling resistance R_0. If R_k is 100 ohms then R_0 will need to be 1,900 ohms. We shall deal throughout this manuscript, however, with the value of R_0' rather than R_0. A value of R_0' of 2,000 ohms may be considered excessive, since the D.C. components of the anode currents of V_1 and V_2 which pass through it may cause the safe heater-cathode insulation rating of the valves to be exceeded. If the total cathode current of each valve is 40 mA and if R_0' is 2,000 ohms, then the steady P.D. across R_0' is $2{,}000 \times \frac{80}{1{,}000}$ = 160 volts. R_0' will, therefore, need to be capable of dissipating steadily a power of $\frac{80}{1{,}000} \times 160 = 12.8$ watts, and if the valves require an anode potential of 250 volts,

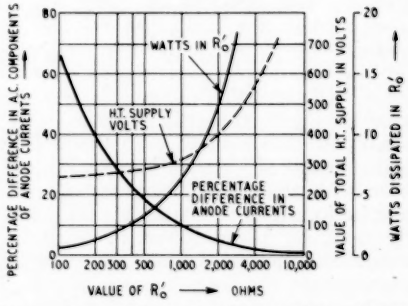

Fig. 2. *Curves illustrating the variation of H.T. supply volts, the percentage difference in anode currents and the power in R_0' with variation of R_0'. These results apply to valves with g_m = 10 mA/volt and total cathode current = 40 mA per valve.*

the total H.T. voltage necessary will be 250 + 160 = 410, which may be considered excessive. The dependence of Δ, power in R_0' and H.T. voltage on the value of R_0' are illustrated in Fig. 2, evaluated for

valves with $g_m = 10$ mA/volt and total cathode current equal to 40 mA per valve. From this a most suitable value for R_0' would appear to be about 1,000 ohms. It would be good enough, probably, to make $R_0 = 1{,}000$ ohms (a standard value), R_k being 100 ohms (a typical value) so that $R_0' = 1{,}100$ ohms. Clearly, for a given value of Δ, in order that the power wasted

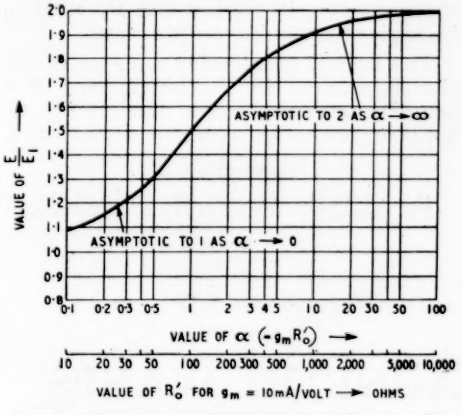

Fig. 3. *Dependence of E/E_1 on the value of $g_m R_0'$.*

in R_0' shall be a minimum and also for minimum value of total H.T. supply, the highest possible value of g_m is necessary. Also, since the input signal for the second valve is given by the product of R_0' and Δ (see equation (2)) it is a good thing to use valves requiring a small input signal, for these will require a smaller value of R_0' for a given Δ than other valves requiring bigger input signals. Pentodes or tetrodes, then, are more suitable to use in this particular circuit than triodes.

Input Signal Requirements

From (1) and (7), we have

$$E_1 = E - \frac{g_m R_0' E}{1 + 2 g_m R_0'}$$

$$= E\left(\frac{1 + g_m R_0'}{1 + 2 g_m R_0'}\right)$$

$$\therefore \quad \frac{E}{E_1} = \frac{1 + 2 g_m R_0'}{1 + g_m R_0'}$$

Now the fraction $\frac{E}{E_1}$ gives the value of the total input signal across AB (Fig. 1) over the input signal required by V_1. Let $g_m = 10$ mA/volt and $R_0' = 1{,}000$ ohms as before. We have

$$\frac{E}{E_1} = \frac{1 + 2 \times \frac{10}{1{,}000} \times 1{,}000}{1 + \frac{10}{1{,}000} \times 1{,}000} = \frac{21}{11} \approx 1.91$$

so that the input signal required by the circuit is, in these circumstances, 1.91 times that required by one of the push-pull valves. Since i_1 is greater than i_2, it follows that V_1 has a larger input signal than V_2 and so overloads first. Suppose V_1 can accept a maximum input signal of 8 volts peak value. The input necessary to the circuit is $8 \times 1.91 = 15.28$ volts for maximum output power. The dependence of $\frac{E}{E_1}$ on the value of $g_m R_0'$ is illustrated in Fig. 3, from which it can be seen that the greater $g_m R_0'$ is the more nearly will E approach twice E_1.

Dependence of Output Power on Value of $g_m R_0'$

It is also interesting to find the effect of varying R_0'. on the power output of the two valves. Since i_1 exceeds i_2, it follows that if V_1 is driven to the limit of its power output, then V_2 is being under-run to an extent governed by the value of R_0'. The power actually delivered by the two valves

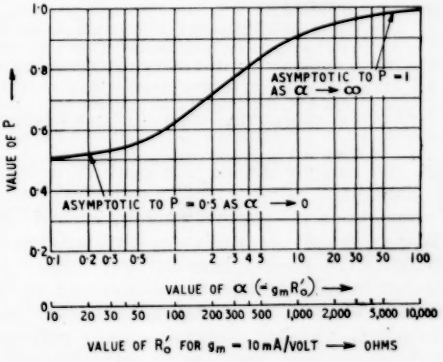

Fig. 4. *Illustrating the dependence of output power ratio on the value of α.*

is proportional to $i_1^2 + i_2^2$ whereas if both anode currents equalled i_1 the power would be proportional to $2 i_1^2$. Hence the fraction of the maximum possible power output actually obtained, P, is defined by

$$P = \frac{i_1^2 + i_2^2}{2i_1^2}$$

$$= \tfrac{1}{2}\left(1 + \frac{i_2^2}{i_1^2}\right)$$

From (5) and (6)

$$\frac{i_2}{i_1} = \frac{g_m R_0'}{1 + g_m R_0'}$$

$$\therefore \quad P = \tfrac{1}{2}\left[1 + \frac{g_m^2 R_0'^2}{(1 + g_m R_0')^2}\right]$$

$$= \tfrac{1}{2}\left[1 + \frac{\alpha^2}{(1+\alpha)^2}\right]$$

where $\alpha = g_m R_0'$

In Fig. 4 this fraction has been plotted in the form of a curve and from this we can estimate the value of α necessary to give the desired power output. Suppose, for example, we want at least 90 per cent. of the maximum possible power output. For this value $\alpha = 10$ from Fig. 4, so that, if $g_m = 10$ mA/volt for the particular valves used $R_0' = \frac{\alpha}{g_m} = \frac{10}{\frac{10}{1{,}000}} = 1{,}000$ ohms. Clearly the greater g_m is the less will R_0' need to be to give the required output power.

This method of achieving push-pull operation is covered by British Patents Nos. 492,407 (General Electric Co., Ltd.) and 508,697 (British Broadcasting Corporation).

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MICRO-ELECTROMAGNETIC WAVES*

The Nature of Spark-Generated Waves

By M. G. Kelliher and E. T. S. Walton

(Trinity College, Dublin)

SUMMARY.—A description is given of a reliable, sensitive, and fairly selective detector for the micro-electromagnetic waves produced by sparking between short tungsten rods under oil. It has usually been assumed that the radiation produced in this way consists of a damped train of waves emitted every 0.01 sec, if the transformer used to produce the sparks is run on 50 c/s A.C. The present experiments show that about 20 to 30 wave trains are emitted every 0.01 sec and that these trains show very much smaller damping than is predicted by theory. It is suggested that some mechanism may be present tending to maintain the oscillations.

IN the early years of this century a considerable number of theoretical and experimental papers were published on the nature of the radiation emitted when sparks were passed between two rods immersed in oil. References to some of these will be found in a paper published by Pollock.[1] The best known work is perhaps that of Nichols and Tear,[2] although only very seldom is any reference made to it in modern books dealing with micro-wave work. A few years ago, Melloh[3] showed that the damped trains of waves produced by the spark method may be used for quantitative work on their transmission through hollow metal tubes. He found that the wave length of the radiation emitted was about 2.5 times the total length of the two rods used, but that the ratio increased considerably when the wave length was below about 5 cm. The factor of 2.5 agrees with the figure 2.53 found theoretically by Macdonald[4], but disagrees with the factor calculated by Abraham[5] of $2\,(1 + 5.6\epsilon^2)$, where $1/\epsilon = 4 \log_e 2a/b$, $2a$ being the total length of the rods and b being their radius. Rayleigh[6] also showed that the factor is nearly 2. Page and Adams[7] derived a complicated formula, the numerical values obtained suggesting that the correction term of $5.6\ \epsilon^2$ in Abraham's formula should be approximately doubled.

The apparatus required for the spark

* MS. accepted by the Editor, August 1945.

method is very simple, and if modified suitably, may be used for producing very short waves. By sparking through a suspension of metallic particles in oil, Glagolewa-Arkadiewa[8] showed that radiation extending into the far infra-red region could be generated. On account of the promise which this method gives for the production of very short waves beyond the range of existing magnetrons and klystrons, a study of the original spark method was undertaken. While writing the present paper, giving the results

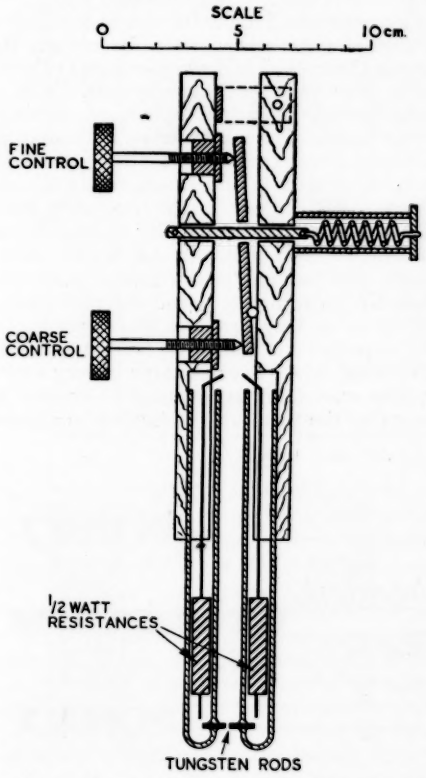

Fig. 1. *The construction of the oil-immersed spark gap is shown here. Resistors are included at the outer edges of the tungsten rods to damp long-wave oscillations.*

of these experiments, a paper has been published by Cooley and Rohrbaugh[9], who describe some new work on the method of Glagolewa-Arkadiewa.

The Generator

The tungsten rods, usually of 2 mm diameter, were sealed into the sides of two tubes of pyrex glass which are immersed in oil. The power is supplied by a small transformer giving about 8,000 volts R.M.S. The current is limited to about 4 mA by two water resistances placed symmetrically between the transformer and the spark gap. Higher currents than this cause rapid wearing away of the tungsten rods, necessitating frequent adjustments to the gap. The output also becomes irregular owing to excessive carbonisation of the oil in the gap. In order to damp out oscillations of longer wavelength which may be radiated from the connecting wires, it has been found advisable to place two $\frac{1}{2}$-watt 10,000-ohm resistances next to the air gaps at the outer ends of the tungsten rods. The connections from these resistances to the water resistances were made of resistance cord.

The function of the air gaps is probably to isolate the tungsten rods from the rest of the apparatus during the oscillations. This is achieved since the transit time of an electron across the air gaps is long in comparison with the period of oscillation. The size of the air gaps is not very important. If the air pressure in them is reduced, the output is found to be roughly proportional to the pressure.

The exact size of the oil gap between the tungsten rods is of great importance, its optimum size varying from about 0.025 mm for an arc current of 3 mA to 0.01 mm for an arc current of 20 mA. A convenient method of making this adjustment is shown in Fig. 1. The coarse and fine controls are well worth the extra complication. Various liquids have been tried for the oil gap, but none has been found superior, as regards size and steadiness of output, to kerosene as recommended by previous workers.

Detection of the Radiation

In our earlier experiments, a simple crystal and point contact connected to a galvanometer were used, the connecting wires in the neighbourhood of the crystal acting as a small aerial. It was found later that the measured value of the wavelength of the radiation emitted depended on the details of the crystal circuit. Hence it was considered advisable to arrange that this could be tuned. The detector shown in Fig. 2 has been found to be very satisfactory. It consists of a concentric-tube type of resonator fitted with a crystal detector (silicon and tungsten point). The silicon is soldered to a

recess in the central rod, while the tungsten wire is supported by a short cylinder which is rigidly attached to the inner rod by some polystyrene and which can slide inside the outer tube without touching it. It thus forms a small capacitor, such as is usual in this type of arrangement. The position of the crystal in the resonator is controlled by the knurled nut N_1 and may thus be adjusted to give appropriate loading of the tuned circuit. The length of the resonator is controlled by the knurled nut N_2 which moves a sliding contact connecting the central rod to the coaxial tube. As this sliding contact is the place of maximum current, it is important that the electrical contact should be a good one. This is secured by making it of double split-collet type, one part of which expands against the outer tube, while the other part contracts on the central rod when the locking nut N_3 is tightened. The contacts with the outer cylinder and with the inner rod are therefore both good ones. The screw S is prevented from rotating by the plate P, and the collet C by a slot and projection arrangement which cannot be seen in the drawing of Fig. 2.

This detector is selective if its fundamental mode of oscillation is used, as the following experiment shows. The wavelength of the radiation from a given oscillator is measured by setting up standing waves formed by reflection at a conducting plane. It is found that the wavelength thus measured depends on the tuning of the detector and not (except in extreme cases) on the length of the oscillator used. The results plotted in Fig. 3 were obtained using a great variety of oscillators, and show a linear relation between the measured wavelength and the length of the detector resonator. The slope of the line is 5.0 instead of the value 4 required if the detector circuit is behaving exactly as a quarter-wave resonator. The intensity for any particular setting of the detector varies greatly with the length of the oscillator. This result shows that the output from a given oscillator is spread over a considerable range of wavelengths.

The presence of higher modes of oscillation in the detector circuit can be shown by passing the radiation down a suitable tube so as to filter out the longer waves. Fig. 4 shows how the measured wavelength depends on the length of resonator, when the radiation from a 1.96 cm oscillator is filtered through an 80 cm length of tube of internal diameter 4.3 cm. AB is the straight line taken from Fig. 3 and CD is obtained by adding to the abscissa of each point on AB a half-wavelength and leaving the ordinate unaltered. Thus CD corresponds to the detector circuit acting as a three-quarter wave resonator, The experimental points lie well on the line CD except when the resonator is very short. In this case the fundamental is excited as shown at the point P. If shorter oscillators and smaller diameter filters are used, the type of oscillation excited in the detector circuit becomes less definite and the tuning of the oscillator has very little effect on the measured wavelength.

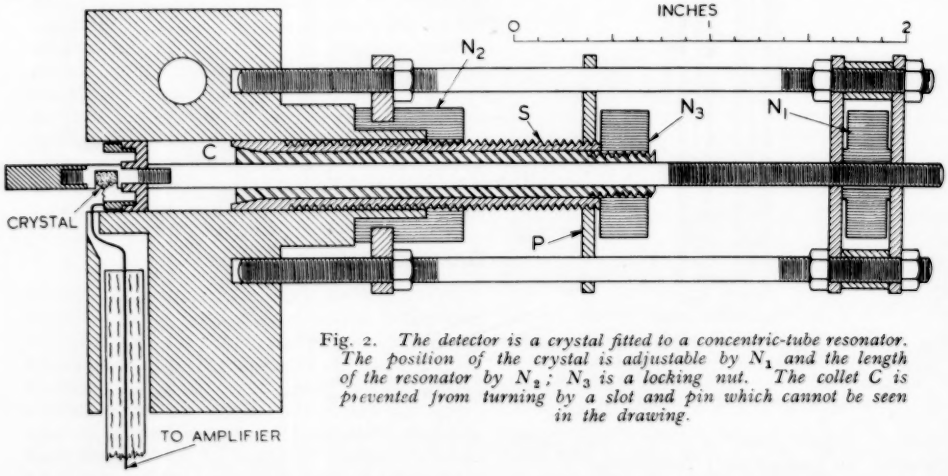

Fig. 2. *The detector is a crystal fitted to a concentric-tube resonator. The position of the crystal is adjustable by N_1 and the length of the resonator by N_2; N_3 is a locking nut. The collet C is prevented from turning by a slot and pin which cannot be seen in the drawing.*

The Amplifier

It was expected that the output from the crystal would consist of one rectified damped train of waves emitted every 0.01 sec. (This view is not correct as will be seen later.) In order to amplify this, a three-stage resistance-capacitance coupled amplifier was used, the time constant of the circuits being about 0.05 sec. The output of this amplifier was fed into a pentode which was transformer-coupled to a full-wave metal rectifier and D.C. milliammeter. This arrangement gives a very sensitive detector with sufficient power to operate a quick-acting robust meter. The overall voltage amplification was about 7×10^6 and the sensitivity could be varied over a wide range by a potentiometer built into the coupling circuit between the first and second valve. As tested by the inverse square law, the output was roughly proportional to the intensity of the radiation over a range of more than 100 to 1, provided the output from the copper-oxide rectifier was not too small.

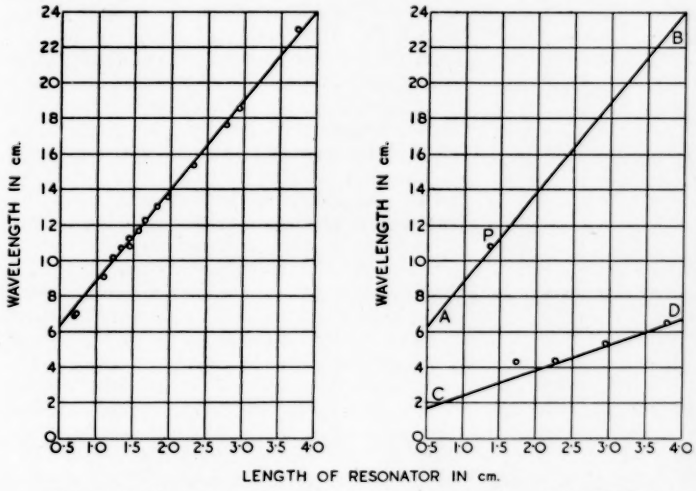

Fig. 3 (Left). *The relation between the wave-length and the length of the detector resonator is linear. In Fig.* 4 (Right) *AB is for the same condition, but CD represents the case when the detector acts as a three-quarter wave resonator.*

Nature of the Radiation from the Oscillator

(1) *Number of wave trains per second.* It has usually been assumed that the radiation from the type of oscillator used in these experiments would consist of one train of damped waves emitted every 0.01 second. To check this, the output from the amplifier (without metal rectifier) was examined using a cathode-ray oscillograph. Fig. 5 shows a typical pattern obtained and is quite different from what one would expect. There are a considerable number of irregularities on the curve, each of these probably corresponding to a train of damped waves. Estimates indicate about 20 to 30 of them every 0.01 second, instead of the one train hitherto assumed. This result was confirmed by examining the oil spark in a rotating mirror, for about 20 to 30 minute sparks were seen to occur every 0.01 second, while the air gaps showed a continuous luminosity over each of these periods. That the radiation was actually being emitted during the greater part of each 0.01 second, was verified by placing between the oscillator and the detector, a disc containing suitably placed holes and rotating it at synchronous speed. In this way, any fraction of the cycle could be picked out and examined separately. It showed that the radiation was being emitted throughout the greater part of each cycle. Thus very little is to be gained by sparking at a higher frequency than is obtained from the ordinary mains, as has been suggested by Hesselbeck.[10]

(2) *State of polarization.* The detector already described is very suitable for determining the state of polarization. The radiation was usually found to be highly polarized, figures as high as 99 per cent. being sometimes attained. The plane of polarization is not always exactly parallel to the tungsten rods. Sometimes it is inclined at angles up to 20°, without there being any obvious reason for it. Refractive indices for materials may be measured readily by finding the angle for minimum reflection and using Brewster's law.

(3) *The energy spectrum.* The wavelength was measured by forming a set of standing

waves by reflection at a metal plane. Fig. 6 gives the usual type of curve obtained. It shows that there are at least 30 waves in the train and that the wavelength can be measured with considerable accuracy. The

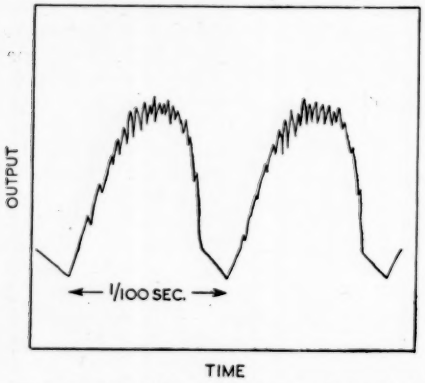

Fig. 5. *This diagram shows a typical radiation wave.*

general rise in the curve can be explained by the increasing inequality in the distances of the detector from the oscillator and from its image in the reflecting plane. Using a given oscillator, a wide variety of wavelengths may be measured in this way by merely varying the tuning of the detector. (See Fig. 4.) This variety must be present in the output of the oscillator, for the production of standing waves could not be explained by assuming that a single wavelength is present and that the tuning of the detector is very broad. This view is supported by the results shown in Fig. 7. The full curve shows how the output of the amplifier depends on the tuning of the detector, the oscillator having a length of 4.5 cm. Remembering that the output of the amplifier is roughly proportional to the intensity, we can calculate the logarithmic decrement from the readings, where the output falls to one half. Its value was found to be 0.61. The dotted line shows the resonance curve calculated for the detector assuming that it acts as a quarter-wavelength concentric-tube resonator loaded by the crystal. The rectifying contact used had a resistance of 3,720 ohms in one direction and 545 in the other, as measured by a Post Office box. This gave a value of 0.28 for the logarithmic decrement. The difference between the two curves in Fig. 7 represents the spread of wavelengths in the radiation. This spread has been confirmed by an analysis of the radiation by filtering it through tubes of various diameters.

The maximum in Fig. 7 occurs at a wavelength of 12.3 cm, giving a value of 2.7 for the ratio of wavelength to length of oscillator. This is higher than even Macdonald's theoretical value of 2.53. It seems reasonable to expect experimental values higher than the theoretical values. In the theory, an isolated oscillator is assumed, while in practice it is supported by pyrex glass and partly surrounded by oil. The presence of these would tend to increase capacitances between various parts of the

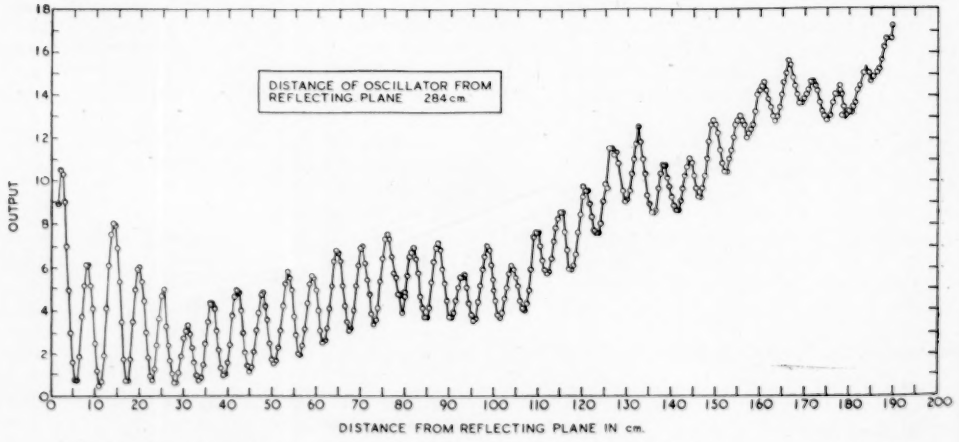

Fig. 6. *Wavelength is measured by forming standing waves, and this diagram shows the type of pattern usually obtained.*

oscillator, and an increase in any capacitance connected to any resonant circuit always tends to increase the wavelength.

(4) *The number of waves in each wave train.* The standing-wave curves show that there are at least 30 waves in a train, and there may be many more. In these experiments the oscillator was allowed to radiate freely

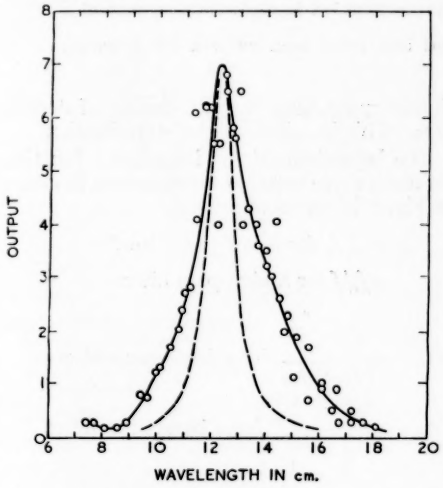

Fig. 7. *Detector Resonance Curves.*

in all directions and was not near any metal which might act as a resonator. Such a resonator, if shock excited, might radiate its energy slowly and thus give a comparatively long train of waves. Various experiments showed that the radiation originated at the oscillator and was not radiated from connecting leads, etc. The comparatively long trains of waves observed are completely at variance with what would be expected theoretically from the radiation damping of the oscillations. Exact calculations for a thin rod do not appear to have been made yet. In the case of a conducting sphere,[11] the logarithmic decrement due to this cause is 3.6. This means that after one complete oscillation, less than 0.001 of the energy remains. Abraham[5] gave for the logarithmic decrement of the oscillations on a thin prolate spheroid, the formula $\delta = 9.74\,\epsilon + 47.6\,\epsilon^2$ where ϵ has the same meaning as given before. This is in close agreement with the results of Page and Adams[7]. For the oscillator used to give the curve in Fig. 6, $2a = 2.4$ cm, $b = 0.1$ cm. Thus $\delta = 1.06$ and the oscillation would be very rapidly damped. While there is some doubt as to how far results for a thin prolate spheroid are applicable to a rod of uniform cross-section,[6] there seems to be no doubt that the order of magnitude is correctly given. It is difficult to explain the absence of rapid damping in Fig. 6, unless we assume that there is some mechanism present which is tending to maintain the oscillations over considerable periods of time. The usual negative-resistance characteristic of an arc may be the agent responsible, or perhaps it may be connected with plasma oscillations studied by Langmuir and Tonks.[12] Whatever may be the explanation, if such a mechanism is present, the spark method of generating very short waves may be of more use in the future than has hitherto seemed likely.

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ERRATA

It is regretted that two errors occurred in "Aerial Impedance Measurements" in the December issue. One of the authors, M. H. Oliver, was described as A.M.I.E.E. instead of Grad. I.E.E., and in Fig. 9 the scale of resistance is shown as 100–1,400 ohms instead of 0–1,300 ohms.

DOUBLE-DERIVED TERMINATIONS*

By R. O. Rowlands, B.Sc.

(The General Electric Co., Ltd.)

SUMMARY.—A method is described for connecting filters in series or parallel such that the real part of their impedance or admittance function is equivalent to that of a double *m*-derived section.

Some of the networks obtained are original and have been covered by a patent application.

Introduction

IT is well known that the image impedance of a constant-*k* type filter varies considerably over the pass range of the filter. When it is desirable that a filter should have a fairly flat impedance characteristic it is generally terminated with one-half

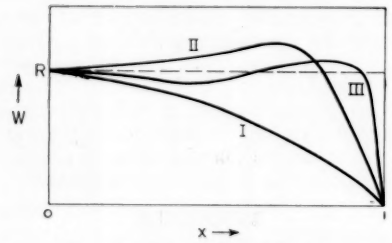


Fig. 1. *Impedance functions with I, Constant-k termination; II, m-Termination; and III, mm'-Termination.*

of an *m*-derived section, a value of $m = 0.6$ being usually chosen. With this value of m the impedance only varies about 5 per cent. over 80 per cent. of the pass range.

Zobel has shown[1] that by repeated derivation the image impedance can theoretically be made to approach a constant value in an oscillatory manner over as large a part of

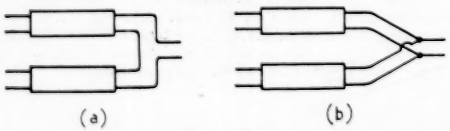


Fig. 2. *Complementary Filters,* (a) *in series, and* (b) *in parallel.*

the pass range as is desired. The higher the degree of derivation, however, the more complicated the filter becomes, so that in practice it is not usual to employ a termination of more than the 2nd degree of derivation. This is called an *mm'*-termination.

The behaviour of the impedance function for these three types of terminations is shown in Fig. 1 in terms of x where

$$x = f/f_c \text{ for a low-pass filter.}$$

$$= f_c/f \text{ for a high-pass filter.}$$

$$= \frac{f - \frac{f_m^2}{f}}{f_{c1} - f_{c2}} \text{ for a band-pass filter.}$$

$$= \frac{f_{c1} - f_{c2}}{f - \frac{f_m^2}{f}} \text{ for a band-stop filter.}$$

Then $0 < x < 1$ represents the pass range, and $1 < x < \infty$ represents the stop range.

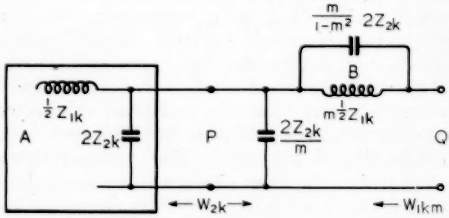


Fig. 3. *A is a composite filter of image impedance* W_{2k}.

Complementary Filters.

These are two filters having substantially the same cut-off frequencies and with the pass range of one corresponding to the stop range of the other. They are connected either in series or parallel at one end, but have their other ends separate as shown in Fig. 2.

The networks (*a*) and (*b*) are inverse networks, i.e. for every property of one network the corresponding property of the other network may be obtained by the application

* MS. accepted by the Editor, October, 1945.

of the principle of duality.[2] In the following discussion, therefore, only one of the above cases will be considered in proving a particular relationship.

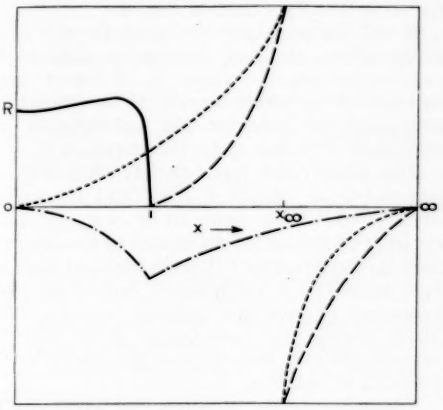
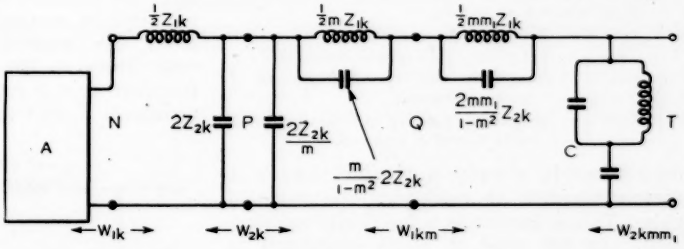


Fig. 4 (Above). *The image impedance of the circuit of Fig. 3 at the terminal Q.*

—— *Real part of* W_{1km}.
– – – *Imaginary part of* W_{1km}.
----- jB.
–·–·– *Imaginary part of* $W_{1km} - jB$.

Fig. 5 (Right). *This circuit is the network of Fig. 3, with a double-derived half-section added at terminals Q.*



Also, although all the formulae will be worked out in terms of Z_{1k} and Z_{2k}, the series and shunt arms of a general filter, the figures will be drawn for the case of a low-pass filter as this will make the argument easier to follow.

Fractional Termination

There are a number of different ways of regarding fractional terminations.[3] For the purpose of this article it is convenient to regard them as being formed in the following manner. In Fig. 3, let A be a composite filter having an image impedance W_{2k} at terminals P. If a shunt-derived half-section is connected in tandem with it the image impedance at the new terminals Q is W_{1km} shown in Fig. 4.

If it is to be suitable for connecting in series with its complementary filter at terminals Q, its impedance in its stop range must be low in order not to interfere with the performance of its complementary filter in the latter's pass range.

The impedance function is resistive in the pass range and reactive in the stop range. It has a pole (i.e. its value is infinity) at one frequency, and so in this form it is unsuitable for connecting in series with its complementary filter at these terminals. The pole, however, is due entirely to the last series arm B of the network, and as B is composed of pure reactances the real part of the impedance looking in at Q is unchanged if B is removed.

i.e. if $W_{1km} = R + jX$
where $R(x) = 0$ for $1 < x < \infty$
and $X(x) = 0$ for $0 < x < 1$
then $W'_{1km} = W_{1km} - jB$
$= R - j(B - X)$

The effect of removing B is shown in Fig. 4. It will be seen that the imaginary part of W'_{1km} is fairly symmetrical about $x = 1$. The complementary filter similarly terminated will have a similar impedance function except that the imaginary part will be of opposite sign, and so when the two are connected in series the imaginary parts of the impedance at the common terminals largely cancel each other out, the extent to which this is achieved depending upon the value of m chosen. The real part of the impedance of each filter will be the same as that of a shunt-derived section.

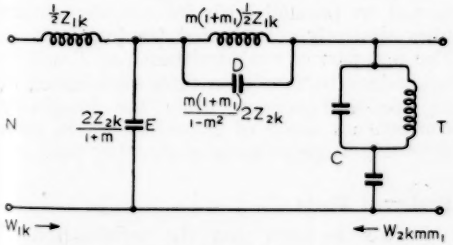


Fig. 6. *The network of Fig. 5 can be reduced to the form shown here.*

The above process amounts to connecting a reactor in shunt with the output terminals of each filter and since the end shunt arm in

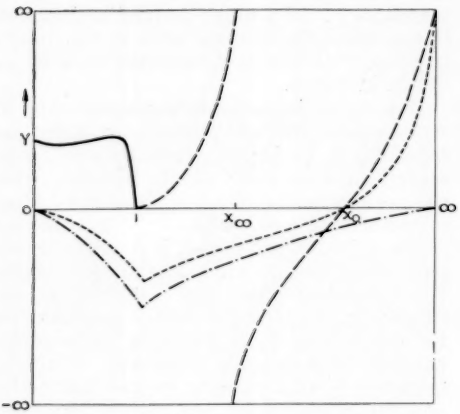


Fig. 7. *The admittance function at terminals T of the network of Fig. 6.*
——— *Real part of admittance at T.*
– – – – *Imaginary part of admittance at T.*
- - - - - - *Imaginary part of admittance at T with C removed.*
—·—·— *Imaginary part of admittance at T with C and F removed.*

each filter is already a similar reactor it merely amounts to modifying the value of this. This is known as a fractional termination. In the case of filters connected in parallel fractional termination consists of increasing the end series arm by $\frac{1}{2}mZ_{1k}$.

Double-Derived Terminations

Having proceeded thus far the next logical step is to attempt to improve the impedance

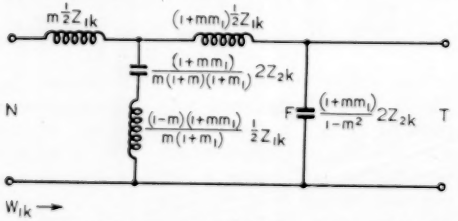


Fig. 8. *The removal of the arm C from Fig. 6 reduces the circuit to this form.*

function still further by connecting a double-derived half-section to terminals Q of Fig. 3 and repeating the method which produced the fractional termination. If the end half-section of A is regarded as being outside A then the network will be as shown in Fig. 5. As we are from now on concerned with the network between terminals N and T, terminals P and Q may be ignored and the network becomes that of Fig. 6.

It will be seen that the network of Fig. 6 differs from that of Fig. 3 in that it is terminated on the right in a shunt arm instead of a series arm. It is therefore necessary to consider the admittance at terminals T rather than the impedance.

This admittance function (shown in Fig. 7) has two poles, one at $x = x_\infty$ and the other at $x = \infty$. The pole at $x = x_\infty$ is due entirely to the series resonance of the arm C, and the removal of C leaves the real part of the admittance unchanged but does not, however, remove the pole at $x = \infty$. (In other words the impedance at T with C removed is still zero at $x = \infty$, due to components D and E).

It can be shown, however (see Appendix I) that the network of Fig. 6 with the arm C removed is equivalent to that of Fig. 8. Here the reactor F $(= D + E)$, which contributes to the pole in the admittance function at $x = \infty$, appears in shunt across terminals T, and so can be removed without

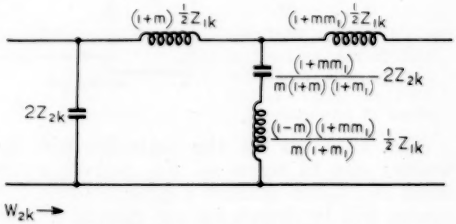


Fig. 9. *The addition of a half-prototype section to Fig. 8 brings the network to this form.*

affecting the real part of the function. The resulting network is the required termination; it can be connected to a filter of impedance W_{1k} at terminals N and connected in parallel with its complementary filter similarly constructed, at terminals T. The real part of the admittance at T will be equivalent to that of a filter terminated in an mm'-half section while the imaginary components being of opposite sign in each filter will largely cancel each other out.

Preferred Form

It will be seen that the termination of Fig. 8 with the reactor F omitted consists of three inductors and one capacitor in the case

of a low-pass filter, and three capacitors and one inductor in the case of a high-pass filter. Since capacitors are in general more economical than inductors, it would be preferable to have an equivalent form of the low-pass termination involving a smaller number of inductors.

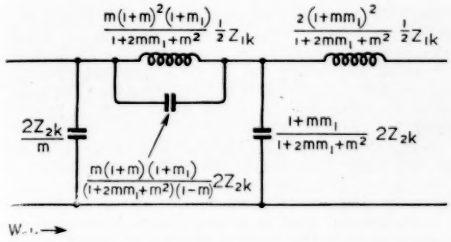


Fig. 10. *This diagram is an equivalent to that of Fig. 9, but requires fewer inductors.*

If half a prototype-section is connected to terminals N of the termination, the network of Fig. 9 is obtained. This can be shown (see Appendix II) to be equivalent to the network of Fig. 10, which fulfils the above requirement.

Acknowledgments

The author wishes to thank a number of colleagues for their suggestions regarding the presentation of the above paper, and the management of the G.E.C. for permission to publish it.

REFERENCES

[1] *B.S.T.J.*, April 1931. The notations used by Zobel will be used throughout the present article.

[2] See e.g. Guillemin's "Communication Networks," Vol. II, page 246.

[3] See e.g. Guillemin's "Communication Networks," Vol. II, p. 356, and Starr's "Electric Circuits and Wave Filters," p. 269.

[4] See Guillemin's "Communication Networks," Vol. II, p. 229.

APPENDIX I

If the above networks are equivalent with respect to their input and output terminals then, using the property that the impedances at these terminals are invariant, we have the following transformation equations in matrix form.[4]

For $\frac{1}{2}Z_{1k}$

$$\begin{Vmatrix} r+t & -t & 0 \\ -t & s+t & 0 \\ 0 & 0 & 0 \end{Vmatrix} = \begin{Vmatrix} 1 & x & 0 \\ 0 & y & 0 \\ 0 & z & 1 \end{Vmatrix} \begin{Vmatrix} c & 0 & 0 \\ 0 & d & -d \\ 0 & -d & d \end{Vmatrix} \begin{Vmatrix} 1 & 0 & 0 \\ x & y & z \\ 0 & 0 & 1 \end{Vmatrix} = \begin{Vmatrix} c+dx^2 & dxy & -dx(1-z) \\ dxy & dy^2 & -dy(1-z) \\ -dx(1-z) & -dy(1-z) & d(1-z)^2 \end{Vmatrix} \quad \ldots \quad \ldots \quad (1)$$

And for $2Z_{2k}$

$$\begin{Vmatrix} p & -p & 0 \\ -p & p+q & -q \\ 0 & -q & q \end{Vmatrix} = \begin{Vmatrix} 1 & x & 0 \\ 0 & y & 0 \\ 0 & z & 1 \end{Vmatrix} \begin{Vmatrix} a & 0 & -a \\ 0 & b & 0 \\ -a & 0 & a \end{Vmatrix} \begin{Vmatrix} 1 & 0 & 0 \\ x & y & z \\ 0 & 0 & 1 \end{Vmatrix} = \begin{Vmatrix} a+bx^2 & bxy & bxz-a \\ bxy & by^2 & byz \\ bxz-a & byz & bz^2+a \end{Vmatrix} \quad \ldots \quad \ldots \quad (2)$$

Equating coefficients we have

From (1) Row 3 $z = 1$

From (2) Row 1 Col. 3 $x = \frac{a}{b}$

From (2) Row 1 $p - p = 0$ gives $a + \frac{ba^2}{b^2} + \frac{bay}{b} = 0$

i.e. $y = -\frac{b+a}{b}$ (3)

so

$$p = \frac{a}{b}(a+b)$$
$$q = a+b$$
$$t = \frac{da}{b^2}(a+b)$$
$$r = c - \frac{da}{b}$$
$$s = \frac{d}{b}(a+b) \quad \ldots \quad \ldots \quad (4)$$

but

$$a = \frac{1}{1+m}$$
$$b = \frac{m(1+m_1)}{1-m^2}$$
$$c = 1$$
$$d = m(1+m_1) \quad \ldots \quad \ldots \quad (5)$$

therefore

$$p = \frac{1-m^2}{m(1+m_1)(1+m)} \times \frac{1+mm_1}{1-m^2}$$

$$= \frac{1+mm_1}{m(1+m_1)(1+m)}$$

$$q = \frac{1-m+m+mm_1}{1-m^2} = \frac{1+mm_1}{1-m^2}$$

$$t = \frac{m(1+m_1)}{1+m} \times \frac{1+mm_1}{1-m^2} \times \frac{(1-m^2)}{m^2(1+m_1)}$$

$$= \frac{(1-m)(1+mm_1)}{m(1+m_1)}$$

$$r = 1 - \frac{m(1+m_1)}{1+m} \times \frac{(1-m^2)}{m(1+m_1)} = m$$

$$s = \frac{m(1+m_1)(1+mm_1)}{1-m^2} \times \frac{1-m^2}{m(1+m_1)}$$

$$= 1 + mm_1 \qquad \ldots \quad \ldots \quad (6)$$

APPENDIX II

If a reactor $e \cdot 2z_{2k}$ is connected across the right-hand terminals of each of the equivalent networks of Fig. 11, then equations (4) remain unchanged except that

$$\frac{1}{q'} = \frac{1}{q} + \frac{1}{e} = \frac{1}{a+b} + \frac{1}{e} \qquad \ldots \quad \ldots \quad (4a)$$

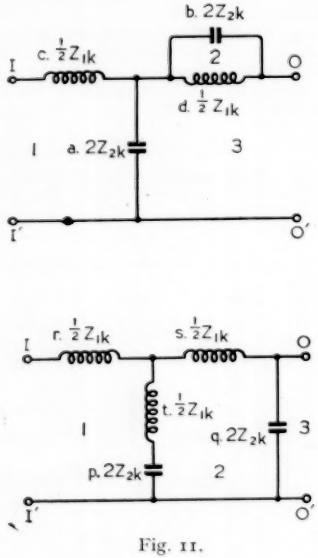


Fig. 11.

and so, solving for a, b, c, d, and e in terms of p, q', r, s and t by means of equations 4 and 4a, we obtain

$$\left.\begin{aligned} a &= \frac{ps}{s+t} \\ b &= \frac{ps^2}{t(s+t)} \\ d &= \frac{-s^2}{s+t} \\ c &= \frac{st}{s+t} + r \\ e &= \frac{pq's}{ps - q't} \end{aligned}\right\} \qquad \ldots \quad \ldots \quad (7)$$

The values of the elements of the network of Fig. 10 can then be obtained from those of Fig. 9 by means of these equations.

CORRESPONDENCE

Pulse Modulation

To the Editor, "Wireless Engineer."

SIR,—The recent proposal by Messrs. Pye, Ltd., of the so-called "videosonic" method of transmitting a television sound channel raises an important issue which appears to be somewhat inexcusably elided in their publications to date; namely, what audio-frequency band does such a system admit?

While the answer to this question is almost obvious for the case of a single sinusoidal component of modulation, it is not so obvious whether any additional restrictions arise when a complex wave form is considered; that is to say, there is no *a priori* guarantee that the system admits of linear superposition. Further, it is not entirely clear whether the audio band restriction has to occur at the transmitter or at the receiver or at both.

In the proposed system, the sound channel is handled by means of a width-modulated pulse transmission, in which the pulses occur during the normal line-sync intervals. Thus the system belongs to a class which may more generally be termed "strobosonic"; the characteristic feature of this class being that the sound waveform is reconstructed from a succession of extremely short glimpses at the original. The signal can thus be conveniently expressed, in terms of a notation which the writer proposed some years ago.* as follows:—

$$\{a_0 + a_1 \cos p_1 t + a_2 \cos p_2 t + \ldots\} \operatorname{col} qt,$$

where the term in brackets represents the complex modulating wave form and col qt is the repetitive impulsive function of afrequency q, having the definition

$$\operatorname{col} qt = 0 \text{ for } qt \neq 2n\pi$$
$$\operatorname{col} qt \to \infty \text{ for } qt \to 2n\pi$$

the nature of the infinity being defined by

$$\lim_{\epsilon \to 0} \int_{n\pi - \epsilon}^{2n\pi + \epsilon} \operatorname{col} \theta \cdot d\theta = 2\pi$$

Using the theorem

$$\cos pt \operatorname{col} qt = \sum_{-\infty}^{\infty}{}_s \cos (p - sq)\, t$$

we see that the signal may be written

$$a_0 \sum_{-\infty}^{\infty}{}_s \cos sqt + a_1 \sum_{-\infty}^{\infty}{}_s \cos (p_1 - sq)t + \ldots$$

The wanted terms in the expression are those corresponding to $s = 0$. All the other terms, which are in the nature of image frequencies with respect to the strobe frequency and its multiples, have to be excluded by filtering.

It is thus clear:

(*a*) That there is an absolute limit for the audio band at half the strobe frequency.

(*b*) That restriction to this band must be carried out by filtering both at the transmitter and at the receiver, and

(*c*) That the linear superposition principle

* *Wireless Engineer*, November 1938.

applies so that no complications arise on considering complex wave forms.

As to practical conclusions, it must be expected that the effective audio band would be considerably below the theoretical limit of half the strobe frequency, since a sharp cut off within the normal audio band is known to be aurally disagreeable, possibly in part due to filter ringing effects. As one is not restricted by expense in the case of the transmitting filter, it may be assumed that the theoretical figure for the audio band can be closely approached; in the case of the receiver, where filtering possibilities are limited, a lower effective cut-off must be expected. Thus for a strobe frequency of 10,125 c/s, corresponding to the present 405-line standard, one may anticipate an audio frequency band of perhaps 4,000 c/s, which seems rather a severe restriction for a service which otherwise could offer the most useful realization of high fidelity.

On the other hand, for an increased line frequency, the system appears to offer attractive possibilities for a sound channel of acceptable quality. The suggestion that there is even room for two sound channels appears unfortunate.

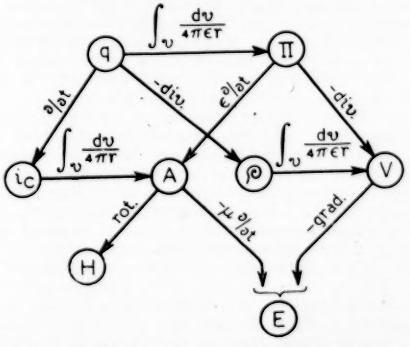
London, N.W.11. L. H. BEDFORD.

BOOK REVIEWS

Antenne e propagazione delle onde elettromagnetiche (per ingegneri)

By BRUNO PERONI. Pp. 372; 119 Figs. Michele dell'Aira—Editore. Rome.

This book is very unusual in that it is not in type but reproduced from handwriting; it is beautifully done and almost as easy to read as if it were printed. The first 58 pages are concerned with the fundamental laws of electro-magnetism and the various potentials and vectors. We reproduce an interesting diagram showing the relation between the various magnitudes. Starting, for



example, at the top left-hand corner with q, the quantity crossing unit area, by applying $\partial/\partial t$ one obtains the conduction current density; by applying $-div$ one obtains the density of charge or any change in it; by integrating $q/4\pi\epsilon r$ throughout the volume one obtains the Hertzian potential Π and similarly for the other magnitudes.

The second chapter which occupies nearly half the book deals with the radiation of electromagnetic waves from aerials under various conditions. Not only are the theoretical formulae developed, but graphical methods are described for the approximate determination of the radiation from actual aerials. Polar curves of radiation distribution are obtained for vertical and horizontal aerials and arrays. The third chapter is devoted to the propagation of waves over the earth's surface and the effects of the ionosphere. Chapter IV deals with receiving aerials, and Chapter V with methods of feeding aerials and aerial arrays, and with directional reception. There are three appendices, the first of 30 pages on vector calculus, the second on vector potential and self and mutual inductance, and the third on the solution of differential equations. The Giorgi system of units is employed throughout.

The scribe has done his work wonderfully well; he makes occasional slips such as resistance in Ωhms and capacitance in Faradays, but the book is remarkably free from such errors. It can certainly be recommended to any advanced student or radio engineer with a knowledge of Italian or the desire to acquire such knowledge.

G. W. O. H.

Annales de Radioélectricité

Vol. 1, No. 1, July 1945. 79 Boulevard Haussmann, Paris.

The associated French radio-telegraphic companies have established at 98 Boulevard Haussmann, Paris, a "Centre d'information et de documentation." Among the activities of this body is the publication of a journal giving an account of the scientific and technical achievements of the associated companies. The new association is really a development of an association of the leading radio manufacturing concerns founded in 1918. It is intended to publish the journal quarterly. The first number contains 80 pages. After an introduction by M. Ponte, a director of the Comp. Générale de T.S.F., there follow four articles; the first by M. Warnecke occupies 48 pages and deals with "La physique et la technique des tubes électronique d'émission dits 'a modulation de vitesse'"; it concludes with a valuable list of 86 references, many of them to French patent specifications. The second paper is by M. Chireix, the chief engineer of the Société Française radioélectrique; it deals with signal and noise power in amplitude, duration and frequency modulation of ultra-short wave pulses. The third article is by M. Laplume and deals with electromagnetic shielding by a tube with its axis perpendicular to the magnetic field. The final paper is a short note by M. Guénard on the maximum current possible in an electronic beam subjected to density modulation along the axis of a cylindrical tube of given dimensions.

This journal promises to make a very useful contribution to the current literature of radio research and development. G. W. O. H.

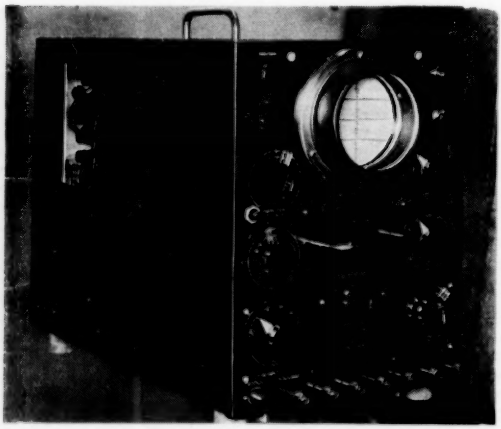
PHYSICAL SOCIETY'S EXHIBITION

Measuring Equipment : Cm-Wave Apparatus : Industrial Electronics

FOR three days in January the Physical Society held its thirtieth exhibition of scientific instruments and apparatus. It was the first to be held since 1939, and was characterized by two things—the overwhelming preponderance of wireless and allied equipment and the enormous variety of such apparatus.

Measuring instruments formed the greater part of this apparatus, and here one of the clearest trends is for the cathode-ray oscilloscope to become a precision instrument rather than a mainly qualitative waveform indicator. It always has been capable of quantitative application, of course, but in the past this often entailed considerable trouble in calibration and the checking thereof. Instruments are now made in which this can be effected simply and quickly by built-in standards.

The Marconi-Instruments Electrostatic Viewing Unit, for instance, is designed especially for pulse work and a " time standard " is included to assist in the measurement of small time intervals. This takes the form of a high-Q circuit which is kicked into oscillation at its natural frequency to produce a damped wave train. Natural frequencies of 50 kc/s, 500 kc/s and 5 Mc/s are provided.



Cossor Model 448 double-beam oscilloscope.

In the Cossor Model 448 double-beam instrument an amplitude scale calibrated in voltage is provided. This oscilloscope is also noteworthy in having a time-base which can be used to give a single stroke, triggered by the input, for use with non-recurring phenomena. It is, of course, usable also as a recurrent time-base. A camera attachment is available.

This firm also produces a multiple-recording unit in which two double-beam tubes and a moving-film camera permit records of four traces to be made simultaneously. 70-mm film is employed and writing speeds up to 3 km/sec. can be handled.

The amplifiers fitted to oscilloscopes now handle a very wide frequency band. In the Cossor model referred to above, the range at low gain is 50 c/s to 5 Mc/s. Mullard have two models; the E805 has an amplifier flat within 3 db from 2 c/s to 2 Mc/s and usable up to 5 Mc/s, while the E800 has a coverage of 0.1 c/s to 40 kc/s with a variation less than 2 db.

Standard signal generators appeared for much higher frequencies than ever before, and the lower-frequency instruments were more refined. The TF 867 of Marconi Instruments, for the range of 15 kc/s to 30 Mc/s, includes an A.G.C. system to keep the output constant within $\pm$ 1 db, as well as the usual manual control for precise setting. A crystal calibrator is fitted, and special steps have been taken to reduce frequency modulation.

A higher frequency model by the same firm (6–300 Mc/s) is of particular interest in that turret-coil changing is used without any switch contacts. Instead of switches, a capacitance connection to the coil is used. Each coil on the turret is provided with two sets of vanes which rotate with the turret and enmesh with two sets of fixed vanes in the " work " position.

At still higher frequencies cavity resonators are used for the frequency-determining elements. Among the types shown by Standard Telephones & Cables was one covering 7.1–11 cm and using a velocity-modulated valve as an oscillator. The frequency is controlled by a plunger moving in a resonant cavity and operated by a micrometer-head type of adjustment.

This firm also exhibited a wavemeter of N.P.L. design, and the National Physical Laboratory itself was showing a similar instrument. This covers 500–10,000 Mc/s with a cavity resonating in a hybrid TM_{010}-coaxial mode. The adjustable plunger is of the non-contact type, and a crystal detector is incorporated. The accuracy is 1 part in 10^4.

Wavemeters for lower radio frequencies were shown by Sullivan. The direct reading Sullivan-Griffiths wavemeter covers 30 kc/s to 60 Mc/s with an accuracy of 1 part in 10^4 immediately it is switched on. Temperature compensation is used, and it is proof against humidity effects.

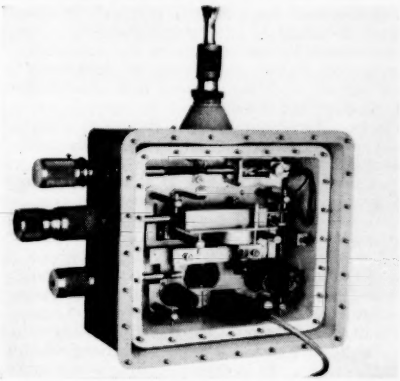
A.F. Oscillators

Among audio-frequency signal sources the beat-frequency oscillator still finds wide application. Mullard showed a model with 4 watts output for 1 per cent. distortion and covering 15 c/s to 20 kc/s. Marconi Instruments exhibited one intended for V.F. work and covering 50 c/s to 5 Mc/s in two bands with an output of 1 watt.

Resistance-capacitance oscillators, however, were

very prominent for purely A.F. applications. The Marconi Instruments model covers 15 c/s to 20 kc/s in three ranges, while Dawe Instruments showed two types. One covers 20 c/s to 20 kc/s with 1 watt output and the other 20 c/s to 200 kc/s with 100 mW output.

The Muirhead-Wigan R.C. oscillator is of especial interest in that the frequency control operates on the decade principle and covers 1 c/s to 111.1 kc/s. There are four decade dials, providing thousands, hundreds, tens, and units, and a key switch giving alternative multiplying factors of 1 and 10. The accuracy of frequency is claimed as $\pm$ 0.2 per cent. or $\pm$ 2 c/s, whichever is the greater.



Standard Telephones and Cables centimetre-wave signal generator

For frequency-measurement at the lower frequencies, Salford were showing a direct reading instrument covering 50 c/s to 45 kc/s which is independent of waveform. The input is converted to a square waveform of constant amplitude and a rectifier stage provides a current proportional to frequency. The reading is stated to be independent of interference provided that the latter is 6 db below the signal.

Impedance measurements were covered by a wide variety of bridges. A Schering bridge covering 0.001 $\mu\mu$F to 4 $\mu\mu$F was exhibited by Sullivan. It is intended for the measurement of valve inter-electrode capacitance with an accuracy better than 0.001 $\mu\mu$F. Muirhead were showing a general-purpose bridge for the measurement at audio-frequency of R, C, L, dissipation factor, and magnification.

Tinsley had a special bridge for use with strain gauges. It is of the unity-ratio type and covers strains from a few parts in a million to 0.5 per cent.

Inductance measurement is carried out with a T-bridge in the Dawe Instruments model covering 10 mH to 100 H. A 1,000-c/s source is used. British Physical Laboratories exhibited equipment designed primarily for production testing. A capacitance bridge, energised at 1 Mc/s, covering 1 $\mu\mu$F to 0.003 μF, an electrolytic-capacitor bridge covering 0.2 μF to 2,200 μF with an automatic overload-protection device, and a Q-meter were the most outstanding examples of their products. The Leland Instruments Q-meter is noteworthy in permitting measurement over the wide range of 30–200 Mc/s.

Measuring Instruments (Pullin) showed a resistance and capacitance bridge covering 1 Ω to 10 MΩ and 10 $\mu\mu$F to 100 μF. A valve voltmeter is included and can be used for external measurements.

The Salford high-voltage valve-voltmeter utilizes the linear relation existing between the grid and anode voltages of a cathode-ray tube at the beam-extinction point. The input is applied to the anode and the grid voltage for extinction is read off a calibrated control. It is suitable for A.F., R.F. and pulses with peak values up to 10 kV.

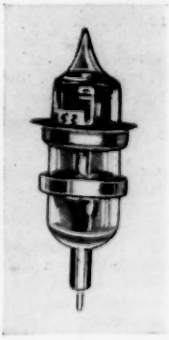
A three-range instrument of the valve electrometer type and covering up to 280 volts was shown by Baldwin Instrument Co. It has an input capacitance of 1 $\mu\mu$F with a resistance of 10^{16} Ω and is due to Dr. F. T. Farmer.

Pointer-type instruments were shown by many firms and two trends are evident. One is towards hermetic sealing and the other towards very high resistance voltmeters. The former trend was well exemplified by the exhibits of Ferranti, Ernest Turner, Elliott and Sangamo Weston and the latter by the multi-range Avometer with a 50-μA movement. This gives a voltmeter resistance of 20,000 ohms per volt. This meter is also interesting in that it incorporates an automatic cut-out to safeguard, not only the meter itself but the shunts.

V.H.F. Equipment

The most outstanding exhibits in the high-frequency field were undoubtedly the valves designed for generating frequencies in the cm-wave region. Both magnetrons and velocity-modulated valves developed chiefly for radar were shown by many firms, and there were also low-power types suitable for the local oscillators of receivers, signal generators, etc. Ferranti, the G.E.C. Research Laboratories and M. O. Valve Co., and Marconi's W.T. Co. showed many types of magnetron, while Standard Telephones & Cables exhibited velocity-modulated valves.

Of almost greater interest were negative-grid triodes for high frequencies. Standard Telephones & Cables showed the CV 127 with an output of 20 W at 600 Mc/s, among others, and the G.E.C., and M.O. Valve Co. had the CV 273 which will oscillate in a concentric-line circuit at frequencies up to 3,500 Mc/s (8.6 cm). This last is of the disc-seal type in which grid and anode are in the form of copper discs extending through the glass envelope and sealed thereto. The connections are made by means of external bands clamped to these discs.

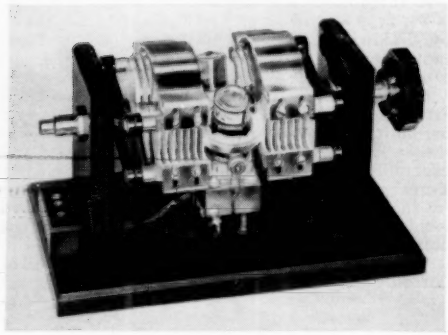


The CV273 disc-seal triode for cm-wave oscillators.

This stand also displayed examples of modern V.H.F. technique in the form of oscillators with line and cavity resonators. Some with a modified form of a normal resonant circuit were also on view, and one, of the butterfly type, had a range of 250–900 Mc/s.

For the measurement of R.F. currents at high frequencies a number of special thermo-couple type detectors are now made. The Cambridge Instrument Co. have a range of vacuum thermo-junctions having cm-wave dipole aerials fixed directly into the sides of the glass wall. Thermo-couples designed for building into the walls of wave-guides and resonators were shown by Standard Telephones & Cables. They are for a current of 15 mA at frequencies of 300—6,000 Mc/s.

Cables with Telecothene insulation were shown in a wide range by T. C. & M. Co. The patterns range



Butterfly oscillator shown by G.E.C.

from low-power 70-ohm feeders of the coaxial type to shielded twin-cables, and there are also some semi-air-spaced varieties.

British Insulated Callender's Cables were showing a similar range of high-frequency cables with poly-ethylene dielectric. With an impedance of about 72 Ω and a capacitance of 22 $\mu\mu$F/ft, the C.W. 95 has an attenuation of 8 db per 100 ft at 1,000 Mc/s. It is a lead-alloy sheathed cable with an outside diameter of 0.45 in and a power rating of about 120 watts.

This firm was also showing waveguides. There are flexible sections built of copper braid and also of gauze, but the most interesting is one which consists of an assembly of metal discs in a moulded-rubber hose. The hose has internal ribs which serve to position and space the metal discs and itself provide the flexibility. The discs are pierced with a rectangular hole in the centre, so that each disc forms a short section of a rectangular waveguide. The dimensions of the spaces between successive discs is such that they form cavity resonators of low impedance at the guide end. The low impedance forms a virtual short circuit between adjacent sections and the guide as a whole behaves substantially as if it were of continuous metal construction.

Industrial Electronics

Many firms were exhibiting apparatus for the measurement of the moisture content in grain in both mains and portable battery-operated types. In all cases a measured quantity of grain is inserted in a cell and some moisture-dependent electrical property is measured. Three scales—for wheat, oats and barley—are usually provided.

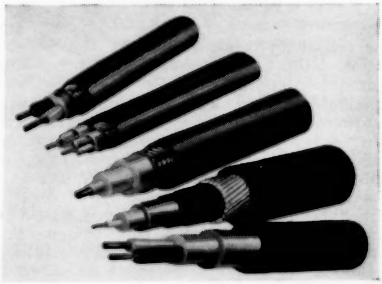
In the Dawe Instruments' model the alternating current passed by a given quantity of grain is measured, and this principle is also adopted for this firm's "Moisture in Timber" meter. Measurement is effected in the Marconi Instruments' model by determining the effect of the sample upon the Q of a circuit. The Mullard instrument, however, measures the capacitance of a grain cell.

The National Physical Laboratory was also showing a device of this type and in this a source of alternating current is applied through an adjustable phase-shifting network to the grain cell and a resistor in series. The phase of the voltage across the resistor is compared in a valve circuit with that of the supply.

The detection of cracks in metals can be carried out in several ways. One method, adopted by Salford, depends on the interruption by a crack of eddy currents in the surface of the specimen. The interruption causes a change of frequency of a tuned circuit, lighting a lamp and indicating on a dial the depth of the crack.

"Echo depth-sounding" is used for crack detection by Henry Hughes & Son. Pulse technique is adopted with 2.5 Mc/s quartz crystals mounted on the surface of the material to act as transmitter and receiver. The signals are displayed on a cathode-ray tube and fault detection is possible at depths up to 12ft.

pH-meters were shown by a number of firms. The Marconi Instruments' model embodies an electrometer triode with a stabilized H.T. supply. The Muirhead model is designed for small and constant input current so that the voltage drop in the electrode resistance is independent of the pH of the solution. It is direct-reading and there is an internal standard for checking the calibration.



B. I. Callender's Cables exhibited screened R.F. cables with polyethylene insulation.

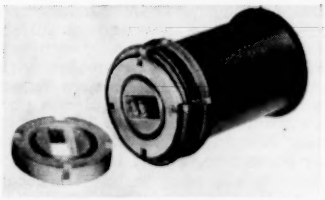
Potentiometer-titration equipment employing a "Magic Eye" instead of a pointer-type indicator was exhibited by Griffin & Tatlock and also by Mullard.

An automatic balance potentiometer is a feature of the pyrometer shown by Honeywell-Brown. The unbalanced voltage is amplified and converted to A.C. It is then applied to a reversing motor which drives the potentiometer in the direction of balance.

Dielectric heating apparatus was by no means unrepresented. The Scientific Acoustics model has an output of 2 kW at 45 Mc/s from a single oscillator, while Ferranti-Wild-Barfield use a push-pull circuit to give 2 kW at 35 Mc/s and their equipment

includes an automatic power output control in which a motor is employed to vary the spacing of the work electrodes. Rediffusion were demonstrating their R.H.2 generator with apparatus for drying powders.

Electro-medical equipment was featured by Marconi Instruments, who showed a theracoupler



B. I. Callender's Cables flexible waveguide.

with an output of 250 W at 10 Mc/s. diathermy apparatus in two types, one providing 350 W at 15 Mc/s and the other 300 W at 50 Mc/s. There was an X-ray dosemeter in which the amplified output of an electrometer triode is applied to an integrator, and a four-channel electro-encephalograph.

The Cambridge electro-cardiograph employs a string galvanometer and a continuous paper camera, and is of remarkably small dimensions.

The electron microscope, with a magnification of 50,000 times, and a resolving power fifty times as great as an optical instrument, was shown by Metropolitan-Vickers. It is of the continuously pumped type operating at 50 kV and includes camera recording equipment. There is provision for visual observation.

Components, Accessories and Materials

The importance of guarding against the effects of humid atmospheric conditions has led to the adoption of hermetic sealing in many components. Quartz crystals in evacuated glass envelopes, similar to that of a valve, were shown by Marconi's W.T. Co., and by Standard Telephones & Cables.

Muirhead had a range of sealed capacitors and the Telegraph Condenser Co. were showing several varieties. The "Metalmite" range comprises miniature types in which the outer aluminium tube is spun on to synthetic rubber end-plugs to form a seal. It is rated to withstand a temperature range of $-40°$ to $+100°$ C with relative humidities up to 100 per cent.

In some cases glass-metal seals are being adopted and the Telegraph Construction & Maintenance Co. were showing Telcoseal No. 1, an alloy having a controlled coefficient of thermal expansion almost identical with that of borosilicate glass. The seals shown consist of a lead-through wire fitted with a glass bead to the outside of which is sealed a flanged metal ring. The flange can be soldered to the metal case of a component so that the unit provides a sealed lead-out wire.

Even when complete sealing is not adopted, some form of tropic-proofing is common. Thus, Standard Telephones & Cables were showing a moving-coil microphone in both tropical and non-tropical forms. The latter has an aluminium-wire coil, but copper is used in the former which has also a lacquer seal. The sensitivity is of the order of -85 db below 1 volt/dyne/cm.

Westinghouse have applied hermetic sealing to small copper-oxide rectifiers and have also improved the moisture imperviety by eliminating graphite as a contact material.

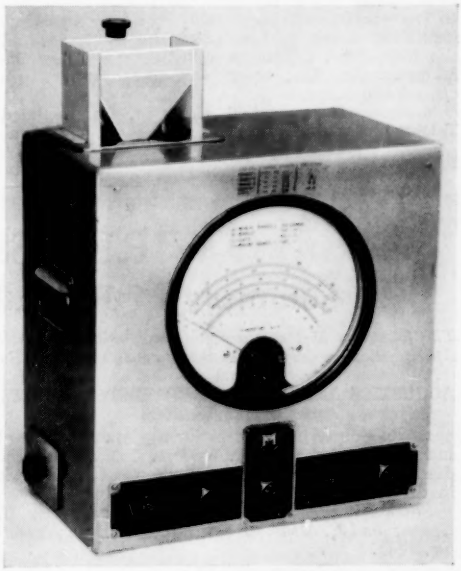
This firm also showed selenium-type rectifiers for power purposes—a type of rectifier also exhibited by Standard Telephones & Cables.

Voltage control and stabilization devices were to be found in considerable numbers. Westinghouse showed the "Noregg" D.C. and "Stabilistor" A.C. stabilizing devices. The latter produces a constant voltage output of good waveform without moving parts.

Zenith had variable voltage transformers with manual or motor control and Gresham Transformers were showing a multi-tapped transformer with a separate commutator-type switch giving an output variable from 0 to 230 volts.

Selenium barrier-layer photo-cells in a wide range of low-voltage types were exhibited by Evans Electroselenium. A high-voltage type, consisting of a battery of single cells, is intended for use with a valve amplifier and the output across $1M\Omega$ is of the order of 1 volt for an illumination of 5 ft candles.

Apart from the V.H.F. types, there were not many new valves to be seen. Ferranti had a cold-cathode gas-triode intended for use with a photo-cell. It needs an input signal of 5 volts, but takes



Marconi Instruments moisture-in-grain meter.

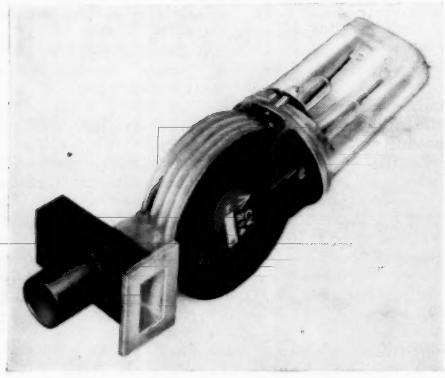
less than 4 μA current, and can control a peak current of up to 20 mA.

This firm also showed electrometer valves, including a double-triode with an ion-screen between the filament and the grids. It is intended for use in balanced circuits.

Cossor exhibited a range of battery valves of miniature type with the ring-seal and miniature

7-pin base. Prototypes of pressed glass-base valves suitable for television frequencies were shown by the M.O. Valve Co.

Mechanical relays and mercury-switches were to



Marconi-Osram CV 214 3-cm magnetron.

be found on a number of stands. They ranged from power types through the telephone-pattern to an aerial-switching relay with polysterene insulation shown by Londex on the one hand, and the high-speed Carpenter relay of the Telephone Manufacturing Co., which will operate up to 300 c/s, on the other.

Materials shown by T. C. & M. Co. included Telcothene in the form of flexible sheet as well as rod, tube and strip and Telcuman. This last is a precision resistance alloy with a resistivity of 42–46 microhms/cm³ and an E.M.F. against copper of less than 1 μV/° C.

The Permanent Magnet Association were showing Alcomax II, a nickel-aluminium-cobalt-iron alloy with a magnetic energy nearly three times as great as Alnico.

The Decca Navigator is intended for course and position finding in aircraft and in surface ships. It is characterized by remarkably compact equipment and, since it operates by measuring the phase difference between continuous-wave transmissions from spaced transmitters, it requires a remarkably small spectrum in the ether. The transmitted frequencies are such that they have common-frequency harmonics, and it is the phase of these harmonics which is compared in the receiver and used to activate a series of direct-reading dials.

Hearing-aids with amplitude compression in the middle and low frequencies were shown by Multitone Electric. This is done in order to reduce masking effect of high intensity sounds and improve intelligibility. One instrument measures only $5\frac{3}{4}$ in $\times$ $2\frac{7}{8}$ in $\times$ $1\frac{1}{2}$ in and weighs $12\frac{1}{2}$ oz, including layer-type batteries.

G.E.C. Research Laboratories had on view a set employing the ASAC (automatic selection of all channels) system. Any one of 336 channels in a band at about 3 metres can be selected. Of the three crystals used, one controls an oscillator the harmonics of which govern the channel spacing. When a channel is selected by means of numbered dials, the band is swept by a motor-driven capacitor and the harmonics operate a counter circuit which stops the motor when the required channel is reached.

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 1/- each.

ACOUSTICS AND AUDIO-FREQUENCY CIRCUITS AND APPARATUS

571 676.—Sintering process for the manufacture of magnetic nickel-cobalt alloys, particularly for the diaphragms of telephones and other electro-acoustic instruments.

Standard Telephones and Cables Ltd.; B. B. Grace; and E. Mills. Application date 19th November, 1943.

571 778.—Condenser type of microphone in which a flat diaphragm is "domed" away from the back-plate prior to the application of the polarising voltage.

O. K. Kolb. Application date 10th December, 1943.

571 870.—Single-diaphragm microphone, the back-plate being formed with central apertures and with peripheral air-passages to produce a uni-directional effect.

O. K. Kolb. Application date 10th December, 1943.

571 963.—Generating low-frequency pulses, as used for code signals, by the control of an oscillating column of mercury.

The Siemens and General Electric Railway Signal Co., Ltd. and H. J. Napier. Application date 21st December, 1943.

AERIALS AND AERIAL SYSTEMS

572 085.—Quarter-wave high-impedance terminating element for the coupling-end of a concentric transmission line feeding a dipole aerial.

The British Thomson-Houston Co., Ltd. Convention date (U.S.A.) 16th March, 1943.

DIRECTIONAL WIRELESS

571 239.—Course-indicating system in which one radiated field is non-directional, whilst the other is directional and is reversed in phase as it swings through the course-line.

Soc. Industrielle des Procedes Loth. Convention date (The Netherlands) 23rd June, 1939.

RECEIVING CIRCUITS AND APPARATUS

(*See also under Television*)

571 314.—Frequency-discriminating circuit comprising a filter of the bridged-T or parallel-T type for receiving frequency-modulated signals.

Philco Radio and Television Corp. (*assignees of R. B. Albright*). *Convention date* (*U.S.A.*) *24th November*, 1942.

571 404.—Receiver in which interstation-noise is suppressed by the action of a multivibrator circuit which is effective only in the absence of a desired signal.

The British Thomson-Houston Co. Ltd. Convention date (*U.S.A.*) *30th January*, 1943.

571 452.—Short-wave adaptor which also acts as an aperiodic amplifier when long or medium waves are being received.

J. Lips. Convention date (*Switzerland*) *10th November*, 1942.

571 458.—Compact variable-resistance device, particularly suitable as a volume-control for a radio receiver.

The Morgan Crucible Co., Ltd., and L. W. Miller. Application date 22nd February, 1944.

571 483.—Gas-filled relay for recording signals consisting of the positive and negative half-cycles of a sinusoidal train.

Western Electric Co., Inc. Convention date (*U.S.A.*) *6th August*, 1942.

571 580.—Super-regenerative amplifier for use as an amplitude-limiter and as a detector for phase- or frequency-modulated signals.

Marconi's W.T. Co., Ltd. (*assignees of G. C. Sziklai*). *Convention date* (*U.S.A.*) *15th December*, 1942.

571 787.—Permeability-tuning device for a short-wave superheterodyne in which two coils are wound with varying pitches to ensure a constant frequency-difference throughout the tuning range.

Marconi's W.T. Co., Ltd. (*assignees of W. F. Sands*). *Convention date* (*U.S.A.*) *24th October*, 1942.

571 836.—Permeability-tuning system for a multi-band short-wave receiver in which an auxiliary continuously-adjustable core is used for band-spreading.

Marconi's W.T. Co., Ltd. (*assignees of W. F. Sands*). *Convention date* (*U.S.A.*) *24th October*, 1942.

571 858.—Permeability-tuning system in which an auxiliary magnetic core is provided for coarse and fine control, and for band-spreading, on each of the short-wave settings.

Marconi's W.T. Co., Ltd. (*assignees of W. F. Sands and J. J. Brand. Convention dates* (*U.S.A.*) *24th October and 21st November*, 1942.

571 898.—Wave-band or like change-over switch, wherein a moving member slides in a trough-shaped fixed member having multiple contacts along its sides.

Marconi's W.T. Co., Ltd., E. R. Burroughes; and H. H. Lightfoot. Application date 30th November, 1943.

572 138.—Silicon-carbide and wire combination for rectifying oscillations of very high frequency by presenting a minimum capacitance-shunt.

The British Thomson-Houston Co., Ltd. and T. H. Kinman. Application date 6th March, 1942.

572 214.—Pair of valves having a common cathode feedback-resistance and arranged to amplify selectively a predetermined band of frequencies.

Cinema-Television, Ltd. and S. S. West. Application date 16th December, 1943.

572 216.—Cascaded valve circuit with selective feed-back for selectively amplifying a predetermined band of frequencies.

The Mullard Radio Valve Co., Ltd. and C. L. Richards. Application date 28th December, 1943.

572 220.—Powder-cored inductance coil in which the core and coil are connected to two lead-in wires which also serve as supports, say from the chassis of a wireless set.

Neosid, Ltd. and M. Grenly. Application date 23rd March, 1944.

TELEVISION CIRCUITS AND APPARATUS

FOR TRANSMISSION AND RECEPTION

571 776.—Means for suppressing the cathode-ray beam on the return stroke of the scanning sweep in a television system.

Standard Telephones and Cables, Ltd. (*assignees of P. S. Christaldi and J. R. Banker*). *Convention date* (*U.S.A.*) *21st January*, 1943.

571 944.—Cathode-ray interlaced scanning-systems for the stereoscopic projection of televised signals, or of a recorded cinema film.

The British Thomson-Houston Co., Ltd. and A. P. Castellain. Application date 1st June, 1943.

TRANSMITTING CIRCUITS AND APPARATUS

(*See also under Television*)

571 225.—Modulation system in which an ionizable gas is used to vary the resonance frequency of an oscillator tube of the rhumbatron type.

W. W. Groves (*communicated by N. G. Schonander*). *Application date 1st October*, 1943.

571 840.—Frequency-modulating system depending upon the use of a resistance-reactance oscillator and a delay-network for frequency-control.

Standard Telephones and Cables, Ltd. (*communicated by International Standard Electric Corporation*). *Application date 11th February*, 1944.

571 970.—Visual indicator for the remote tuning-control, say of the tank circuit of a wireless transmitter.

Standard Telephones and Cables Ltd. (*assignees of H. R. Sherwood*). *Convention date* (*U.S.A.*) *13th February*, 1943.

572 006.—Means for controlling or stabilizing the frequency of the tuned circuits in frequency-modulated transmitters, or in superheterodyne receivers.

"*Patelhold*" *Patentverwertungs &c. Akt. Convention date* (*Switzerland*), *19th May*, 1942.

572 161.—Holder or mount for one or more transmitting valves with provision for the supply of cooling-fluid.

Standard Telephones and Cables, Ltd. (*assignees*

of R. B. Hoffman). Convention date (U.S.A. 24th June, 1942.

572 259.—Metering arrangement for detecting and measuring the modulation of carrier-oscillations with respect to the amplitude, or instantaneous frequency or phase, of the oscillations.

Marconi's W.T. Co., Ltd. (assignees of M. G. Crosby). Convention date (U.S.A.) 24th March, 1943.

CONSTRUCTION OF ELECTRONIC-DISCHARGE DEVICES

571 187.—Construction of the base and contact-pins of a miniature valve with a so-called " button " stem.

Standard Telephones and Cables, Ltd., and S. J. Powers. Application date 19th November, 1943.

571 431.—Construction of the metal base of a valve of the type to which a screening can be permanently secured.

The M–O Valve Co., Ltd., and C. W. Cosgrove. Application date 6th August, 1941.

571 503.—Cathode assembly and construction designed to generate a narrowly-defined election stream, and to be free from certain undesirable heat-expansion effects.

Marconi's W.T. Co., Ltd. (assignees of W. M. Trumbell and L. F. Coombs). Convention date (U.S.A.) 29th September, 1942.

571 678.—Construction, and use as an amplifier, detector, or generator, of an electron discharge device designed to produce a split beam of electrons.

G. R. E. Cleveland. Application date 20th January, 1944.

571 709.—Valve cathode designed to be operated at high voltage and under a pressure different from that prevailing in the main bulb.

Westinghouse Electric International Co. Convention date (U.S.A.) 17th October, 1942.

571 764.—Construction and arrangement of the cathode starting device in a gas-filled discharge tube of the " make-alive " type.

Westinghouse Electric International Co. Convention date (U.S.A.) 7th May, 1942.

572 000.—Gas-tight articulated or ball-and-socket joint, to allow the electrode of an electron-discharge tube to be adjusted externally.

Standard Telephones and Cables, Ltd. and R. N. Hall. Application date 28th January, 1944.

572 257.—Construction and arrangement of the anode of a high-tension rectifier.

The Mullard Radio Valve Co., Ltd., O. Pressel, and G. A. M. Diepstraten. Application date 22nd February, 1944.

SUBSIDIARY APPARATUS AND MATERIALS

571 290.—Photo-electric relay for stabilizing the intensity of aviation and other navigational signalling-lights against fog and other atmospheric variations.

Chance Bros., Ltd., and J. G. Holmes. Application date 8th September, 1943.

571 294.—Installation for testing or adapting radio apparatus to withstand extreme variations of temperature, pressure and humidity.

Air Control Installations, Ltd., and G. E. Clifford. Application date 4th October, 1943.

571 345.—Thermionic-valve converter-unit for supplying high-frequency current from direct or A.C. mains to an ultra-violet therapeutic lamp or other gaseous discharge device.

H. W. K. Jennings (communicated by Sun-Kraft, Inc.). Application date 23rd July, 1943.

571 498.—Dielectric containing one or other of the aromatic sulphones as an ingredient to give a high S.I.C. and low power-loss.

The British Thomson-Houston Co., Ltd. Convention date (U.S.A.) 31st March, 1942.

571 599.—Holder or mount for resiliently clamping a thermionic valve or other electrical component having a frangible base.

Belling and Lee, Ltd., and E. M. Lee. Application date 18th June, 1943.

571 623.—Method of mounting and exciting a piezo-electric crystal of the A.T.-cut type so as to allow slight deliberate variations of the fundamental frequency.

The General Electric Co., Ltd., and S. K. Lewer. Application date 24th February, 1943.

571 652.—High-frequency transmission line consisting of a number of conductors held together, but spaced apart, by a helically-intertwined strip of insulating material.

Marconi's W.T. Co., Ltd. (assignees of W. R. Koch) Convention date (U.S.A.) 27th February, 1943.

571 656.—Arrangement for preventing sparking across the moving contacts of an electric current converter.

The General Electric Co., Ltd., and E. Friedlander. Application date 21st April, 1942.

571 687.—Apparatus of the teleprinter code type for indicating at a distance say the bearing of an aeroplane as determined by a D.F. installation.

Standard Telephones and Cables, Ltd., G. C. Hartley, E. M. S. McWhirter, and J. Handley. Application date 11th March, 1941.

571 772.—Artificial line made of strips of high-permeability metal, interleaved with insulation, and provided with a number of intermediate tapping points.

Standard Telephones and Cables, Ltd. (assignees of L. A. de Rosa). Convention date (U.S.A.) 8th August, 1942.

571 798.—Construction of fixed capacitor to reduce heat expansion and to improve the power-factor. (Addition to 552 707).

Sir H. Ingram, Bart. Application date 3rd January, 1944.

571 817.—The use of supersonic sound-waves for the detection of hidden flaws say in a pneumatic tyre.

Wingfoot Corporation. Convention date (U.S.A.) 2nd September, 1942.

571 905.—Process for removing traces of iodine from the surface of a selenium rectifier, in order to increase its " reverse " resistance.

W. W. Triggs (communicated by Bolidens Grov. Akt). Application date 12th January, 1944.

572 018.—Valve-holder, with spring clip to take the valve-pins, and an associated lower tab to which the circuit connection is soldered.

British Solenoids, Ld., S. J. Tyrrell, and A. C. Young. Application date 7th October, 1943.